Structured Flows Across Collective Dynamics and AI

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In July–August 2022, an algal bloom on the Oder River resulted in an environmental disaster. It caused severe damage to the river's ecosystem, health issues among the local population, and (inevitably) mild tremors in the local political landscape. A simple collective action of unicellular organisms led to their aggregation and rapid reproduction, resulting in massive, complex macro-scale repercussions—a vivid example of what scientists refer to as emergent phenomena in *collective dynamics*.

Modern science increasingly shows that very different systems related to collective dynamics—from flocks of birds and swarms of insects to neural networks and artificial intelligence algorithms—can be described using a similar, unified language. A key concept is the notion of a *flow*, representing the evolution of a system over time. The mathematics of flows allows us to capture both how individual particles or agents move and how the entire collective evolves.

In recent years, *Wasserstein flows*—flows in the abstract space of probability measures—have gained particular importance. They provide a powerful framework for studying phenomena across multiple scales, from microscopic interactions to large-scale biological, social, and computational processes, each imposing additional geometric structure to the flow.

The project aims to develop and apply the theory of such flows in four main areas:

- **Interaction-driven dynamics:** Understanding how groups of agents move under mutual forces. The project's key aim is to incorporate the problematic singularity of the interactions into the geometry of the space in the hope of facilitating the description of the related phenomena.
- **Heterogeneous gradient flows:** Relevant in biology and ecology, where different populations may exchange mass (such as migration of individuals between communities). The goal is to adapt recent frameworks to models of multiple species of interacting agents.
- **Transformer architecture:** Chat GPT and other large language models can be represented as *transformer models* of collective dynamics governed by relatively complex differential equations. The goal is to capture the macroscopic evolution of transformer models using simpler, low-dimensional descriptors.
- **Flow matching of heterogeneous data:** Flow matching, including stochastic interpolation, plays a vital role in AI-based image generation. In many applications, it is desirable to control specific components of the image (e.g., eyes, hands) via their own target distributions to reduce artifacts such as extra fingers or misplaced limbs. The goal of the project is to perform flow matching of such heterogeneous data using fibered Wasserstein flows.

By bridging these perspectives, the project demonstrates that a single mathematical structure—the flow—can serve as a *common language* for describing both natural and artificial systems, provided its underlying structure is suitably adapted. This approach opens the door not only to a deeper understanding of collective phenomena in nature but also to new tools in artificial intelligence.