The central focus of this project is to understand how certain symmetries from number theory, known as Galois actions, interact with deep geometric structures that emerge when Calabi–Yau spaces undergo degeneration. Calabi–Yau manifolds are geometric objects that lie at the intersection of many areas of mathematics and physics. They are defined by two key features: they support a "volume" form that never vanishes, and they exhibit a form of cohomological simplicity. These properties make them central in fields ranging from algebraic geometry and complex analysis to number theory and even theoretical physics, most notably in string theory, and more recently in models describing gravitational interactions between two black holes.

Our approach to studying Calabi–Yau manifolds is through a method called degeneration. This means we examine families of such spaces where most members are smooth and well-behaved, but some special members become singular, that is, they develop certain kinds of geometric "defects". These singular fibers often encode rich information about the entire family.

We investigate two distinct scenarios of degeneration.

In the first, we consider a family of Calabi–Yau manifolds varying over a complex disk, where one central fiber becomes singular. Associated to this setup is a monodromy operator, a kind of transformation that captures how the geometry of the smooth fibers changes as one loops around the singular point. A key aim here is to understand how this monodromy relates to the structure of the center fiber, and in particular, how it influences the way Galois groups act on the limiting mixed Hodge structure, a sophisticated algebraic gadget that keeps track of how the geometry degenerates.

In the second case, we move from the complex world to the arithmetic setting. Here, we consider Calabi–Yau manifolds defined over p-adic rings, number systems that arise naturally in number theory. Again, we study degenerations, but this time in mixed characteristic (where the geometry has both "characteristic 0" and "characteristic p" features). Our goal is to understand how the geometry of these degenerations influences certain number-theoretic objects, namely the Euler factors of L-functions, which encode arithmetic information about the manifold. A particularly exciting direction here is the search for new examples of Calabi–Yau manifolds in characteristic p that cannot be lifted to characteristic 0, a phenomenon that suggests truly new arithmetic behavior.

Throughout the project, we will work with two families of concrete examples: Schoen's fiber products, and certain double covers introduced of 3-dimensional projective space. These examples provide fertile ground for exploring the intricate connections between geometry, number theory, and algebra.