This research project aims to explore and utilize the connection between Description Logic and a specific type of logic games. **Description Logic** (DL) is a formalism used to represent, analyze, and reason about structured knowledge, often captured in the form of knowledge bases, ontologies, or taxonomies. The beginnings of DL date back to the early 1990's, when the first systems for automated reasoning over formalized information were developed. DL captures knowledge through *concepts*, which are constructed in a disciplined manner from basic concept names using logical constructors.

Today, large systems of interconnected concepts, definitions, statements describing various knowledge domains have been created. These systems, known as *ontologies*, often rely on the formal tools provided by DL and are thus referred to as *DL ontologies*. Most important examples of such ontologies are: SNOMED CT, `https://snomed.org/` with more than 360 000 concept definitions, Galen, `https//www.opengalen.org/` with more than 23 000 concepts or Foundational Model of Anatomy, `http://si.washington.edu/projects/fma` with more than 100 000 concepts. These and more than 1200 other ontologies are available at BioPortal, `https://bioportal.bioontology.org/`.

DL offers a suite of reasoning services that support the development and maintenance of ontologies. For instance, DL algorithms can be used to verify consistency, detect implicit relationships between concepts, and enforce desired properties, such as disjointness between certain concepts. The present project aims to advance these services by improving the underlying algorithms that enable such reasoning tasks.

One of the most critical tasks in DL applications is subsumption checking—determining whether one concept is more general than another. Subsumption can be computed efficiently when the set of logical constructors used in concept formation is restricted. In such lightweight or fragmented description logics, subsumption can often be decided fast (in polynomial time). However, for more expressive logics, subsumption becomes computationally harder—sometimes requiring exponential time to decide.

The complexity of reasoning tasks like subsumption not only depends on the expressiveness of the logic but also on the presence of background knowledge. Ontology engineers often introduce a set of axioms that provide foundational assumptions for inference. In DL terminology, such a set of axioms is called a TBox (terminological box).

In this work, we focus on the restricted description logic $\mathscr{FL}_0$. The subsumption problem in $\mathscr{FL}_0$ is solvable in polynomial time. However, when a set of axioms (a TBox) is introduced, the complexity increases significantly, making the problem ExpTime-complete. It was observed that there are connections between solving the subsumption problem in $\mathscr{FL}_0$ modulo a TBox and solving a certain type of logic games, called pushdown games. This idea was used to establish ExpTime-hardness of subsumption in $\mathscr{FL}_0$ modulo a TBox, by reducing solving of a pushdown game, which is itself ExpTime-hard problem to the subsumption problem.

We aim to continue this line of research, by demonstrating that this class of pushdown games is equivalent to the problem of subsumption in $\mathscr{FL}_0$ modulo a TBox. Furthermore, we intend to extend the notion of logic games to the problem of *concept unification* in restricted description logic such as $\mathscr{FL}_0$.

Concept unification is another important reasoning service proposed for ontology maintenance. When two or more concept definitions in an ontology describe the same phenomenon, they are considered redundant. Eliminating such redundancy may not only simplify the structure of large ontologies and facilitate their maintenance, but can also improve the efficiency of query answering, particularly in Ontology-Based Data Access (OBDA) systems.

For example, the concept *fracture of femur* may be defined as: "Fracture of lower limb and all values of the finding site property must be instances of the femur structure." In an alternative extension of the same ontology, an engineer might define the same disorder as: "Fracture of bone, where all values of the finding site property must be instances of the lower limb and the femur." These two definitions can be shown equivalent if we assume that "fracture of lower limb" is defined as "Fracture of bone, where all values of finding site are instances of lower limb" and that "femur structure" is semantically equivalent to "femur".

The concepts in this example can be formalized in the language of $\mathscr{FL}_0$. For such unification problem instances, we have an algorithm capable of deciding unifiability. Moreover, the PI of this project (with co-authors) has developed an application, FILO, which implements unification for $\mathscr{FL}_0$. Nevertheless, these developments can become truly useful for the aforementioned applications only if we are able to extend the results to more expressive logics.

In this project, we aim to focus our research on the logic $\mathscr{FL}_0$ extended with a TBox. The decidability of unification in this setting remains an open question. We conjecture that, due to the close connection between subsumption in $\mathscr{FL}_0$ extended with a TBox and pushdown games, the unification problem in this setting can be approached through techniques developed for solving such games.