

Popular science description of the project entitled  
"DISJOINTNESS AND ERGODICITY OF LOCALLY HAMILTONIAN FLOWS AND  
THEIR EXTENSIONS"  
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The aim of this project is to investigate so called flows on surfaces. These are objects describing various physical processes, often associated with the movement of bodies. An example of such processes is ex. the journey of planets around the sun, the dynamics of gas particles or the movement of tectonic plates. Modelling of these processes with help of mathematical notions is one of the oldest problems in science and today we all utilize the achievements of mathematicians and physicists, who worked on progress in this branch of mathematics.

In mathematics the research very often boils down first to investigating simplified models, which are then perturbed to obtain a picture closer to the physical description. One of such simplified models, in which I am interested, is the so-called *billiard in a polygon*. Imagine a billiard table in the shape of any polygon with holes in each of its corners. Assume that we hit a ball and it moves with a uniform speed on our table (it is a simplification, since in reality there exists a friction which slows down the ball). As in the classical billiard, the ball reflects from the edges of our table by the rule "angle of incidence is equal to the angle of reflection". Moreover, if it falls into the hole, its movement finishes. Such system composed of the table, the ball and the movement is called a *billiard flow*.

Imagine now three persons observing the billiard table with a moving ball on it. One person stands by the table, another stands 5 meters further and the third observes the table in a reflection of a Christmas ornament. The picture seen by each of those persons is of course different. However, each of them views exactly the same table! To illustrate, each observer will now exactly in the same moment when a ball reflects from the edge or fall into a hole. One of the subjects that interests me is how to distinguish whether two pictures describe the same system and when completely different. As exceptionally surprising and non-intuitive example of two such non-similar systems, I investigate flows coming from billiards, in which the ball moves exactly along the same trajectories, but with different speed. Similarly, we may imagine, that one part of the billiard table is very hot, while the other is very hot. The flow extensions, which appear in the title of this project, are exactly the way to describe how the temperature of the billiard ball oscillates along its trajectory.

Billiard flows are not the only objects that I investigate. The same questions can be asked regarding other flows of physical origin. However, they are a perfect illustration of what I am planning to work on during the duration of the project. I deeply believe that my research will help understand better unpredictability of processes of this kind.