Polishability of analytic equivalence relations and generalized Scott analysis

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We most often associate mathematical research with applications of mathematics in various other fields of science and technology. Mathematics can serve us to model phenomena and processes, to find optimal solutions. Such mathematical considerations are carried out within a variety of mathematical structures. Thus, for example, we describe time with the real line, computer or logistic networks are simply graphs for mathematicians, and we look for solutions to differential equations in certain Sobolev spaces. From the point of view of theoretical mathematics, it is interesting to study the properties of these abstract structures in isolation from the real phenomena they describe. It is the *abstraction* that allows us to solve real problems with theoretical tools.

To organize the study of such abstract structures, the concept of isomorphism (Gr. *isos* - equal, *morphe* - shape) is extremely useful. Informally: two structures of the same type (e.g., two groups) will be called *isomorphic* if they are indistinguishable from the point of view of the theory of a given type (e.g., group theory). For example, the finite group of integers $\{0, 1, \ldots, 10, 11\}$ with addition modulo 12 is isomorphic to the multiplicative group of complex elements of the equation $x^{12} = 1$. Using such a group, we can, for example, describe the passage of hours on the face of a clock, but we also use such groups for more complicated applications (for instance, in banking, we use modulo 97 arithmetic to detect errors in IBAN account numbers).

Depending on what structures we consider, the concepts of isomorphism also differ: the isomorphism relation of groups and the isomorphism relation of Banach spaces are two different mathematical objects. The research we are conducting is concerned with the classification of equivalence relations (in particular, isomorphism relations) using the

(i) *invariants*,

e.g., abstract structures such as finitely generated Abelian groups are classified up to isomorphism by quite uncomplicated objects: finite sequences of integers (Theorem of Frobenius and Stickelberger, 1878);

(ii) *Borel reducibility*,

which allows one to order relations according to their (properly defined) level of complexity.

More precisely, our research is concerned with the properties of polishable equivalence relations, a notion introduced by S. Solecki. For a polishable equivalence relation, there exists a certain procedure that approximates this relation. Similar procedures are products of generalized Scott analysis for equivalence relations induced by continuous actions of Polish groups and for isomorphism relations of separable metric structures of a fixed language. We want to investigate the precise correspondence between these notions. Moreover, we want to investigate the polishability of a certain relation, for which Scott analysis cannot be carried out. A positive answer would indicate that the procedure generated by polishability is more universal than Scott analysis. Our another goal is determining whether polishable equivalence relations are classified by orbit equivalence relations of continuous actions of Polish groups.