# The quest for smoothness in complex geometric analysis

The general opinion on what mathematcis *is*, apart from trivial calculus, is that mathematics boils down to *solving equations.* Indeed, in geometric analysis significant number of problems can be rewritten in the form of a PDE - examples include *minimal surfaces*, *metrics with prescribed curvature* and many more. Quite contrary to common beliefs though even writing a problem abstractly in a form of equation as

$$P(u) = f$$

(here $u$ is the unknown $P$ is an abstract operator and $f$ stands for the pregiven data) is not the **end** but rather the **beginning** of the solution. This is because solving such abstract "*equations*" not only requires sophisticated tools but in many cases there are no solutions or at least no solutions with the expected **regularity**. Thus a lot of research in geometric analysis falls within the following scheme:

(1) If possible solve $P(u) = f$ and then investigate additional properties of the solutions such as uniqueness, stability and exact dependence on the initial data $f$;
(2) If solving $P(u) = f$ is not possible directly then try to enlarge the space of *solutions* by introducing potentially singular ones. Then if a solution $u$ is found study what kind of singularities occur and what are the circumstances of their formation.

Typical mathematical questions arising in Step 2 of this scheme are as follows:

- Understand which data $f$ lead to singular solutions and for which no singularity occurs;
- Understand the singularities of the solutions themselves i.e. is the solution smooth away from some small set? What kind of structures the set of singular points can form?
- Understand how can on **rule out** the existence of singularities regardless of the initial data $f$.

Classical example of such a scheme is the investigation of *minimal* surfaces and the associated *Bernstein problem.*

In the current project we want to study various problems arising in complex geometric analysis and understand the formation of singularities. Our set of problems include, but is not limied to, studying elliptic nonlinear PDEs on complex manifolds.