Quantum GTA I: Groups, Topology and Algebras

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In the proposed research project we intend to investigate subtle properties of operator algebras, focusing on examples related to quantum symmetries.

Let us briefly sketch the background of the proposal. Noncommutative phenomena have played a prominent way in mathematics and in mathematical physics since the discovery of quantum mechanics in the first half of the 20th century. Formally this has resulted in the birth of theory of operator algebras, natural objects combining analytic and algebraic aspects, which represent ensembles of quantum observables. The theory has grown enormously in the last almost 100 years, unveiling often unexpected connections to many classical fields of mathematics such as dynamical systems, probability theory, theory of information or group theory.

On the other hand the group theory, in particular in its modern guise of *geometric group theory*, viewing collections of symmetries as geometric objects themselves, has in the last twenty-thirty years had a great impact on the study of operator algebras. New examples and questions have changed the way we think about many of the fundamental problems, and relate to fields such as quantum information. But the story does not end with classical symmetries: mathematical formalism and physical intuitions have led to creating the notion of *quantum groups*, expressing quantum symmetries.

Our primary goal will be to investigate the approximation properties of operator algebras arising from quantum symmetries. By approximation properties we understand subtle analytic or topological ways in which a given complicated structure may – or may not – be modelled by much simpler objects. In the context of our project such properties have deep roots in Fourier analysis. The latter has revolutionised mathematical analysis by expressing very general functions as limits of trigonometric polynomials, which are very well understood. The idea of investigating such phenomena will be the guiding light throughout the project. Their very nature requires a special care in understanding how exactly the 'approximation' should be understood topologically – which explains the T in our GTA acronym.

Many of the problems are formulated in an open-ended, speculative way, but they are always motivated by very concrete research questions. In particular in the last part of the project we will investigate the consequences of our abstract results for quantum probability and quantum information, two modern and rapidly developing fields of mathematics, with very concrete and often immediate connections to computer science and theoretical physics. Long term perspective is related to smooth incorporation of quantum group examples in recent fascinating discoveries in the theory of operator algebras, related to classification and rigidity of the structures in question.

The research will be undertaken by a team based in Warsaw, but will also involve several contacts with a wide network of world-based experts with ties to the PI. Specialists working on quantum groups, operator algebras and quantum information theory, but also mathematical physicists will profit from our interdiciplinary approach. It will both lead to abstract discoveries and expand our understanding of quantum mysteries from a mathematical perspective.