METRIC ALGEBRAIC GEOMETRY OF COMPRESSED SENSING
ABSTRACT FOR THE GENERAL PUBLIC

One of the most urgent requirements of today is a reconciliation between environmental and economic aspects of modern society, and this attention should also be applied when handling and acquiring digitalized data. A sustainable approach to data management is provided by **compressed sensing**, an engineering technique that reconstructs the full data from only partial measurements of it. An instance of this technique is reconstructing a complete picture starting from a partial one where, for instance, some of its pixel are not visible.

A key feature of data is their structure, and data coming from different context can be formulated in a similar fashion if one recognizes the common complexity underlying them. For instance, the movie-recommendation problem, where suggestions are made to the users based on the features they love the most, and the real time cardiac MRI problem, a type of medical image reconstruction, can both be mathematically modeled and solved in the same way.

However, fundamental questions relating to the resource-efficiency of this recovery process remain:

1. Is it possible to increase the resource efficiency of recovery techniques by exploiting other key data features such as their **invariance under relevant transformations**? Imagine to perform the reconstruction of a picture. In the recovery process, it does not matter if the picture is rotated by 90 degrees, right? Perform recovery in the context of structured data exhibiting invariants is the content of my **first objective**.

2. Is it possible to characterize algebraically the **ill-posed locus** of compressed sensing problems, i.e. where the reconstruction problem has no longer a locally continuous solution? Such locus contains instances for which it is impossible to recover a *unique* solution. This encodes the **second objective** of the proposed plan.

3. Is the data reconstructed from partial measurements close to its exact value even if the initial data was slightly perturbed (e.g., by noise)? It is crucial to **quantify the expected sensitivity** of compressed sensing techniques to such perturbations. This is the **third objective** of the present research.

To answer the aforementioned questions and develop the corresponding **fundamental research**, I will combine compressed sensing with metric algebraic geometry and numerical methods.

At its core, algebraic geometry solves systems of polynomial equations and algebraically formalizes the space of their solutions, which are called algebraic varieties. Metric algebraic geometry is a recent trend emerged to satisfy the necessity of answering questions that relate the metric of a proposed model. Blending numerical methods to tackle and assist the resolution of more theoretical problems is becoming more and more present in the recent years.

Hence, the proposed project brings together different disciplines that will all benefit from the performed research both short and long term, so the project will have high scientific impact. The plan is to establish a common language that will bring new inputs for both algebraic and engineering communities, creating new research markets that will affect several applied sciences. The proposed research potentially has an economic and technological impacts that go beyond the duration of the project. This comes from the overarching motivation of all objectives to economize resources use. Compressed sensing is the basic technology underlying several real life applications and being able to reduce the number of measurements will have an impact on the quality of service for these technologies in the precise moment they start being used.