## Quantum switch with continuous control: from foundations to advantages in quantum information

Quantum theory and general relativity are the most successful physical theories so far. However, a "quantum gravity" theory involving both of them has never been formulated. An interesting approach to quantum gravity is the field known as *indefinite causal order*, which puts together the probabilistic character of quantum mechanics and the dynamical causal structure of general relativity. The basic idea of indefinite causal order can be better understood through its most prominent example: the quantum switch. Consider two transformations U and V which change the state of a quantum system. We can apply these transformations in two different orders, either first U and then V, or first V and then U. But in quantum mechanics a quantum system can be in a superposition of two different states, which is like "being in two states at the same time"; this counterintuitive idea is well illustrated by the famous Schrödinger's cat, who is in superposition of dead and alive within a box until someone opens the box and looks at it. In a similar fashion, the quantum switch implements the operations U and V in a superposition of two different orders and it is only when someone measures the system that their order may become definite.

Since its introduction, the quantum switch has seen many applications in several areas such as quantum computing, quantum communication, quantum thermodynamics and quantum metrology, outperforming strategies with definite order. One of the most striking applications of the quantum switch is the activation of communication channel capacity, i.e., using the quantum switch it is possible to transmit a larger amount of information through a noisy channel than using channels in a fixed order. The advantages provided by the quantum switch have been demonstrated in many photonic experiments.

Recently, a generalization of the quantum switch, known as *quantum switch with continuous control* (QSCC), has led to new effects such as the *accelerando/rallentando effect* which can be compared to listening to a song in a superposition of different speeds. Surprisingly, this effect appears naturally in some gravitational scenarios where a massive body is in a superposition of different locations.

This project aims to better characterize these new phenomena using the process matrices framework, a mathematical formalism that allows to certify whether a given process is an instance of indefinite causal order. Several strategies based on the process matrices framework have been used to demonstrate that the quantum switch is indeed an indefinite causal order process. However, it has been conjectured that the accelerando/rallentando effect is not captured by the process matrices framework and therefore escapes known certification methods. The main goal of this project is to prove this conjecture and to build a suitable extension of the process matrices framework that fully describes the QSCC. Furthermore, the project investigates potential advantages of the QSCC in quantum information processing. For instance, it is expected that the activation of channel capacity using the QSCC will surpass the capacity achieved by using the traditional quantum switch. The source of that advantage will be investigated as well.

The results of this project will contribute to a deeper understanding of indefinite causal order and its applications and will broaden its scope to new phenomena eventually present in quantum gravity scenarios.