Slicing balls in p-th norms and Khintchine inequalities

Abstract for the general public

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Let us consider a cube of unit side and intersect it with a plane passing through the center of the cube. Depending on the position of this plane the shape of the intersection changes. Which of these sections have the smallest and the largest area? It turns out that the smallest section is a square of unit side and the largest is a rectangle with sides 1 and $\sqrt{2}$. Therefore the area of a central section of the cube is always between 1 and $\sqrt{2}$. The same bounds hold true in the space of dimension n due to the results of Hadwiger (1972) and Ball (1986).

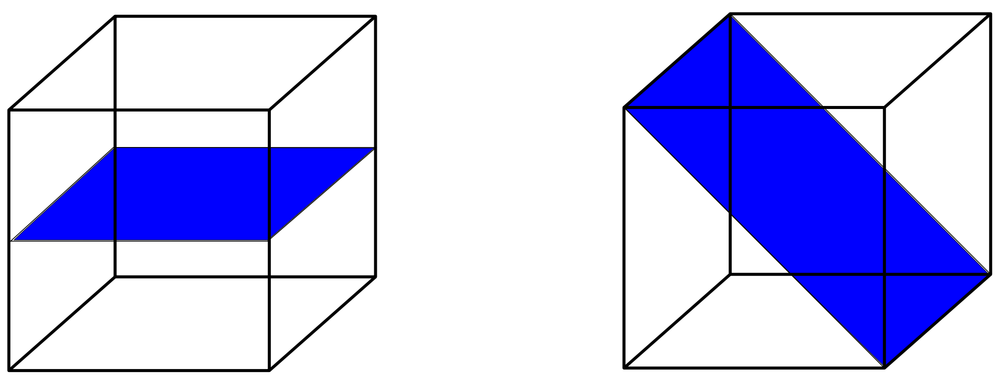

Figure 1: Minimal and maximal section of the cube.

Let us now introduce a more general family of sets, namely sets of points (x, y, z) given by inequalities of the form

$$|x|^p + |y|^p + |z|^p \leq 1.$$

If p approaches infinity, this set becomes very close to the cube of side length two. If $p = 2$ we get the set of points satisfying the inequality $x^2 + y^2 + z^2 \leq 1$, which is the unit ball. In this case all sections are unit discs of area π. For $p = 1$ we get another interesting set, namely the set of points satisfying $|x| + |y| + |z| \leq 1$, which turns out to be the octahedron.

Except for very large p, even in this simple three dimensional situation the largest sections are not known for $p > 2$. One of the main goals of the project is to investigate this problem and its counterparts in n dimensional spaces.