The goal of the project is creating mathematical theory to understand fundamental properties of several models from image processing and solid state physics. Mathematically speaking, we will analyze certain optimization problems in the realm of calculus of variations, and related evolutionary partial differential equations (PDEs).

A unifying feature of considered problems is strong degeneracy and/or singularity, leading to very limited regularity of solutions. They are typically only expected to be *functions of bounded variations* (BV). In particular, they are allowed to have jump discontinuities. In the modeling context this is desirable—think of sharp contours in images, or grain boundaries in polycrystals.

However, from the point of view of mathematical analysis it is a difficulty. As everyone who attended first year calculus course knows, discontinuous functions cannot be differentiable! Thus, the derivatives appearing in investigated differential equations need to be understood in a non-classical weak sense. This need for careful definition and cautious handling of considered mathematical objects is a pervasive feature of this project, and one of the reasons why many of the questions we plan to investigate remain unsolved.

We plan to tackle a variety of problems. Two of them concern well-posedness for certain evolutionary PDEs. That is, we are asking whether a solution to the equation exists (and in what sense), and whether there is only one. This is a fundamental question in analysis of PDEs, without which investigation of further properties does not make sense. The PDEs under consideration are:

- the 1-*harmonic map flow*, a model of denoising of data which are constrained to a surface, such as the color component of an image, orientation data of objects or diffusion tensor imaging data;
- the recently formally derived Liu-Lu-Margetis-Marzuola model of thermodynamic fluctuations in crystal surfaces, which has a very novel structure from mathematical point of view.

We also want to investigate a simplified version of the latter, the *fourth order total variation flow*, which has also been proposed as a denoising model. We are interested for example in the question, whether this equation is capable of preserving the shape of objects in an image.

Another class of problems that we will consider deals with certain functionals, i.e., mappings from a set of functions to numbers. The basic problem is to define the functionals of interest in a proper way on BV functions, and show that the functionals attain their minimal values. This is the fundamental task in the calculus of variations. We plan to carry it out for

- *variable growth total variation*, which was proposed as a regularizing term for an adaptive denoising model;
- a variant of a functional on transformations between two given images proposed by Ball and Horner as a measure of similarity between images in computer vision.