POPULAR DESCRIPTION OF THE RESEARCH PROJECT

The project assumes solving four problems. The first one is related to the real Jacobian hypothesis and assumes finding conditions sufficient for the injectivity of real polynomial maps $f : \mathbb{R}^3 \to \mathbb{R}^3$, which are local homeomorphisms and additionally have a small asymptotic set. This condition is the cohesion of the counter-image of a large sphere by mapping f.

The second goal is to solve the following Ebenfelt-Rothschild problem: Given a germ of a finite holomorphic mapping $f : (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ and the germ of an analytic set X in the target, if the pre-image $Y = f^{-1}(X)$, with the reduced structure is smooth, then so is X.

This interesting problem, posed in 2007, is related to the question whether we can remove the singularities of the analytic set using a finite mapping. We want to solve the problem when X is a complete intersection.

Our next goal is to describe generic polynomial maps from $\mathbb{C}^3$ to $\mathbb{C}^3$. Polynomial complex maps $F : \mathbb{C}^n \to \mathbb{C}^n$ are one of the most basic and important tools in mathematics. Despite this, their topology was not known even in the generic case. In our previous works, we described their topologies in dimension two. Now we want to obtain an accurate description of the singularities of polynomial maps in dimension three. In particular, we want to calculate the degree of geometric locus of each type of singularity or multisingularity.

Counting discrete singularities is crucial to achieving our next fourth goal. We would like to show that if a mapping has the maximum possible number of discrete singularities, it must have a generic topological type. As a conclusion, we would show the topological stability of generic polynomial maps and obtain an effective criterion for checking whether a map has a generic topological type.