

Quantum field theory (QFT) is often regarded as the foundational language of physics. It bridges two previously separate areas: our understanding of space and time in special relativity and the description of physical processes in quantum mechanics. QFT is an incredibly successful framework, underpinning everything from our interpretation of particle collider experiments to the development of advanced materials.

Despite its successes, QFT still faces major unresolved challenges. For instance, it is only well-understood in the **perturbative regime** — a simplified scenario where interactions between particles are weak. Beyond this regime, our understanding remains incomplete. Another critical issue is that QFT, in its current form, appears incompatible with general relativity, our best theory of gravity.

This project focuses on an important class of quantum field theories called **non-linear sigma models**. These models are central to particle physics, general relativity, string theory, and solid-state physics. They are formulated using advanced mathematical concepts — specifically geometric methods — which allow the same model to appear in diverse physical contexts.

However, non-linear sigma models are notoriously difficult to solve. The core challenge lies in their non-linearity, which prevents the theory from being separated into independent “free” (kinetic) and interaction terms. As a result, standard techniques like perturbation theory often fail. Consequently, sigma models are typically well-understood **only in specific cases**, such as in two-dimensional spacetime or when they exhibit high degrees of symmetry.

This project aims to overcome these limitations by studying sigma models that go beyond such restrictive assumptions. Using a combination of established tools (e.g., integrability) and newer approaches (e.g., generalised geometry and non-relativistic limits), the project seeks to explore sigma models in higher dimensions and those without special symmetries. This work is expected to provide new insights into the elusive non-perturbative regimes of quantum field theories, where strong interactions dominate.

In certain cases, sigma models in two dimensions can be **integrable**, meaning they can be solved exactly. This project proposes developing new methods for studying such integrable models, relaxing common assumptions like the need for a large amount of symmetries.

Another powerful feature of sigma models is the existence of **dualities**. Dualities occur when two seemingly different models turn out to have identical physical properties, either at the classical level or even in the full quantum theory. These dualities often link the complex, non-perturbative aspects of one model with the simpler, perturbative aspects of another. However, they only appear in sigma models with specific symmetries. Recent research by the principal investigator (PI) and others has shown that generalised geometry — an extension of traditional geometric methods — provides a framework for understanding these dualities, even in more generic sigma models.

Finally, the project will explore **decoupling limits**, which isolate specific subsectors of a sigma model. These limits often simplify the model enough to study its non-perturbative properties. For example, non-relativistic and ultra-relativistic limits have gained significant attention in the past decade, particularly in two- and three-dimensional sigma models. These approaches challenge the traditional assumption that sigma models must rely on Riemannian geometry — the mathematical framework closely tied to our intuitive sense of shapes and distances. By relaxing this assumption, the project opens the door to exploring new physical and mathematical perspectives on sigma models.