A wave function for an N-site lattice system can be understood as a tensor with N indices, each index with a dimension equal to the physical dimension of a site (e.g. 2 for a spin 1/2). The tensor grows exponentially with N limiting brute force simulations to small N. The key idea of tensor networks is to replace the huge tensor by a network of small tensors, contracted through virtual bond indices, and optimize the small tensors one by one instead of the huge wave function at once. The bond indices' dimension is a refinement parameter that limits the amount of entanglement within the network state. Even for an infinite lattice, there is an important class of quantum states, whose entanglement satisfies the area law, that can be represented efficiently by a tensor network with a finite bond dimension. Thanks to this some long standing problems, not tractable by any other method, were finally settled with tensor networks. They include the nature of the ground state of the notorious/celebrated Hubbard model for strongly correlated electrons hopping on a square lattice that is the simplest model to capture some physics underlying the high temperature superconductivity.

In this project we apply the same strategy to thermal states of quantum many body systems that also can be represented efficiently by tensor networks. The main goal is to characterize a finite temperature phase diagram of the Hubbard model and its effective $t - J$ model and in particular their hypothetical pseudogap and stripy phases. This work will capitalize on our earlier experience with simulation of thermal states, including the Hubbard model at medium temperatures, and some recent technical developments inspired by our competition against quantum simulators. As a positive feedback, our anticipated results will serve as a guide for ongoing efforts to quantum simulate the Hubbard model with ultracold atoms.

We will also continue developing the tensor network toolbox beyond the 2D ground and thermal states. One line of development is a generalization to 3D, where evaluation of expectation values in a tensor network state was identified as the main bottleneck. We plan to speed it up with suitable Monte Carlo sampling. Another line is calculation of spectral functions at finite temperature that will combine our experience with both real and imaginary time evolution of 2D tensor networks. The real time evolution itself will be applied to model recent quantum simulations of time dependent Hamiltonians. Finally, we want to address the long standing technical issue of virtual entanglement loops/strings expected to parasite the limited bond dimension of a genuinely 2D tensor network. This would pave a way towards more efficient simulation of systems with long range interactions, like the Hubbard model with the next nearest neighbor hopping, or with plaquette terms like the lattice gauge theories.