BEYOND THE SZEMERÉDI THEOREM

ABSTRACT FOR THE GENERAL PUBLIC

What will be the next term of the sequence $6, 13, 20, 27$? Questions like this are used to check pattern detection skills; for that reason, they are often used to test mathematical abilities. In the example above, each subsequent term is greater by 7 than the previous one; hence the next term will be 34. Sequences of this form, in which the difference between consecutive terms is constant, are called *arithmetic progressions*.

The search for patterns like arithmetic progressions in subsets of natural numbers is a central task of the subfield of pure mathematics called *additive combinatorics*. It is known that all sufficiently large subsets of natural numbers contain an arithmetic progression of arbitrary length; in fact, they contain plenty of such patterns. This theorem, obtained in the 1970s by Szemerédi, is the cornerstone of additive combinatorics. Although it was originally proved using graph theoretic methods, the Szemerédi theorem has so far received close to 20 different proofs using techniques from various areas of mathematics. Its place at the intersection of various mathematical subdisciplines has long been appreciated; it is for precisely this reason that Terence Tao described it as the "Rosetta stone" of mathematics. The tools developed in various proofs of the Szemerédi theorem played an instrumental role in deducing numerous far-reaching generalisations of the result, and so by now we know many other families of patterns that are guaranteed to appear in all large subsets of integers, primes and other sets of interest.

This project has multiple objectives, all of which build on Szemerédi's theorem and related results. One goal is to detect new patterns in *infinite* subsets of integers. Another goal is to understand how big *finite* subsets of integers can be if they avoid certain natural configurations. These problems can be approached by studying the *structure* of and obtaining *estimates* for various multilinear operators, whether coming from additive combinatorics or ergodic theory. Understanding the analytic properties of such operators, including whether they *converge*, and if so, what is their *limit*, is another objective of this project. These problems lie at the core of a very active research program in combinatorics, dynamics, analysis and number theory, and progress on any of them will give us a better understanding of the structure of large sets and fine patterns appearing therein.