

Beyond Worst-Case Analysis: Online Problems with Delays and Stochastic Arrival Times

In our project, we aim to enhance how online algorithms are evaluated. Traditional methods focus on the worst-case scenarios and often fail to represent the typical, more predictable data behaviours observed in the real world. Here, we intend to make our models more reflective of actual conditions. For this reason, we assume that the data we work with follows some stochastic distribution (in other words, is randomly generated). Our study focuses on deciding the best time to address service requests that accumulate over time, weighing the costs of immediate versus delayed responses. We are developing strategies for complex scenarios like online matching, facility location, and online service, for which the worst-case analysis fails to provide reasonable performance guarantees. Below, we provide straightforward explanations of the problems we are considering.

Online Matching To understand this problem better, imagine an online chess gaming platform that wants to maximize the overall satisfaction of the game for its users. Two main factors contribute to this — for each pair of players, we need to look at the experience difference (the smaller, the better), and at the waiting time to start a game (the shorter, the better). The problem here is that each of these parameters needs to be minimized at the expense of the other. For example, finding an opponent with a similar experience level results in a longer waiting time. Thus, the question arises of how to balance these costs when matching the players.

Facility Location The problem of online facility location with delay involves dynamically determining the most effective locations for deploying multiple temporary service units, such as food trucks, across a city. As requests for services arrive over time from various locations in the city, these units can be temporarily established at any chosen point to meet the needs. Each request carries with it a unique delay function, which provides a measure of urgency or priority based on how long the request has been pending.

The objective is to strategically decide which locations should be activated throughout the day to efficiently manage the varying intensities and timings of requests. Opening a facility incurs a fixed cost, and connecting a request to a facility costs an amount proportional to the distance between the request's location and the facility. The facilities themselves are momentary — they appear to meet the demand at a specific location and, once the needs are addressed, they disappear to relocate elsewhere, eliminating the need for a detailed schedule for each unit. This approach corresponds to a food truck leaving its current location after serving all customers there.

Online Service The problem of online service with delay can be seen as managing a mobile repairman who travels across the city to address incoming repair requests at various locations. Each request arrives over time with an associated delay function, which represents the urgency of the repair based on factors like the severity of the issue or how long it has been waiting to be addressed.

The goal is to strategically determine the most efficient locations for the repairman to visit in real time, ensuring that urgent requests are prioritized and travel is minimized. The repairman appears at a designated spot to meet the specific demand, repairing everything from appliances to electrical issues, and once the work at that location is completed, he moves on to the next request. This strategy requires dynamic decision-making based on the evolving landscape of requests.

While the descriptions provided above offer a simplified overview of the challenges we plan to address, the actual problems we will work on are considerably more complex. We aim to develop deep insights and innovative solutions that will significantly contribute to the area of online algorithms with delays. We hope our research will provide a robust foundation for improving how these algorithms perform in real-world applications.