

## PROBABILISTIC ASPECTS OF FRAÏSSÉ LIMITS

The project is concerned with infinite graphs and their generalizations. A graph is one of the most basic mathematical objects: it is just a collection of vertices with a certain set of connections between them.

One of the most important infinite graphs is the Rado graph, also known as the random graph. The process of its construction can be described as follows: we consider pairs of its vertices one by one, flip a coin and place an edge exactly in the case where a tail falls out. In this way, with probability 1, we get the random graph. The random graph was discovered in the 1960s and has been studied intensively since then.

The main idea we want to develop in our project is to investigate what kind of graphs we will get if we slightly change the above procedure. This time, again, we want to draw edges, except that the coin we use is becoming less and less symmetrical - so that, for example, the probability of throwing out a tails tends to zero. The idea of changing the probability of tails seems natural (we can understand this situation in such a way that over time we become more and more reluctant to draw edges), and it leads to interesting questions. Namely, if the sequence of probabilities of tails goes to zero “not very fast”, we can still get a random graph. In this case, however, everything depends on the order in which we choose the edges to be drawn. It turns out that we can get other interesting graphs when we change the order.

A random graph is an example of the so-called Fraïssé limit, a rather general concept that covers a whole range of universal and homogeneous structures. We want to apply the results of our research also to this more general class with hope of obtaining interesting examples not only of graphs, but also, for example, hypergraphs, partial orders and other mathematical structures.

Our research has a quite surprising connection with the seemingly unrelated issue of measurability of filters on natural numbers. In fact, the idea of changing the way a graph is drawn was inspired by an article by Bartoszyński, who considered the measurability of ultrafilters with respect to measures on the Cantor set derived from the toss of an (increasingly) asymmetric coin.