Real-life statistics of ultracold gases and its consequences

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We are tossing two coins. What is the probability of getting a tail and a head? Anyone can perform this experiment themselves – just toss the coin many times, divide the number of head-tail results by the total number of tosses, and thus estimate this probability. In this way, we will find that getting a head on one coin and a tail on the other happens in 50

However, if we consider both coins indistinguishable, we would treat the outcomes head-tail and tail-head as one possibility. Then we would have to acknowledge that we have three possible outcomes, which would change the answer to the originally posed question.

In the 19th century, Leibnitz stated that indistinguishable objects cannot exist. In our example with the coins – one can always assume that one is on the left and the other on the right, thus distinguishing them. Location is an additional feature making objects distinguishable. Following this line of thought, even identical objects are distinguishable.

Reality turned out to be more surprising than "philosophers dreamt." Indian physicist S. Bose noticed that by assuming the indistinguishability of light particles, i.e., photons, Planck's law can be explained. A. Einstein, inspired by Bose's idea, developed the theory of gases, assuming that atoms are also indistinguishable. His way of calculating probabilities caused shock and criticism among scientists, but soon it turned out that the idea of indistinguishable atoms was correct. Proper mathematical formulation of quantum mechanics allowed breaking away from thinking in terms of particle positions – thus changing the foundations on which Leibnitz relied.

One of the consequences of the indistinguishability of particles and the new statistics is the phenomenon of the Bose-Einstein condensate. The phenomenon is that when a gas is cooled below a certain temperature, a significant portion of the atoms suddenly enters the same state, almost immobile atoms forming a matter wave. In the case of an ideal gas at the limit of temperature approaching absolute zero, on average, all particles form a condensate. The key word is on average – Einstein's formalism would lead us to the conclusion that even at zero temperature, the condensate either does not appear at all or, often quite the opposite, appears with a number of particles greater than the average number of particles in the system. These huge fluctuations, called the fluctuation catastrophe, were pointed out by E. Schrödinger, criticizing Einstein's formalism. Indeed, a better understanding of statistical mechanics shows that condensate fluctuations strongly depend on the adopted model. The final word belongs to the experiment – in 2019, for the first time, the precision was increased so much that measuring Bose-Einstein condensate fluctuations became possible [1]. Another, more accurate measurement conducted on a larger number of cases showed that the fluctuations are smaller than predictions in the canonical ensemble [2]. Both measurements were carried out by Jan Arlt's group at Aarhus University and interpreted within the framework of joint work with groups from Warsaw.

In this project, I would like to study the statistical properties of an ultra-cold gas by analyzing the cooling process itself. I expect that, depending on the experimental procedure, it is possible to generate gas with different statistical properties. I will compare the statistics derived from the gas cooling method with models typical for quantum statistical mechanics. I intend to investigate how the details of the statistics reflect on quantum correlations between atoms, particularly on density fluctuations and quantum coherence in the system. Correlations in particle positions, in turn, translate into the rate of particle losses and the accuracy of estimating parameters characterizing the gas, such as its temperature. In the final stage of the project, I intend to examine how reducing condensate fluctuations translates into its practical application in metrology.

[1] M. Kristensen et al. Phys. Rev. Lett. 122, 163601 (2019).

[2] M. Christensen et al. Phys. Rev. Lett. 126, 153601 (2021).