

Research Project Objectives

The notion of *proof* is integral to the concept of justification in mathematics, and is at the core of Philosophy of Mathematics and Epistemology. However, the concept of *proof provides only an incomplete picture* of mathematical justification: Epistemic practices such as mathematics have *cornerstones*, preconditions or presuppositions, which are essential for the practice's epistemic integrity. That proofs in our best theories can only provide a partial Epistemology is not only a philosophical, but also a mathematical fact: Since the discovery of the *Incompleteness Theorems* due to Kurt Gödel, philosophers, logicians and mathematicians *must face not only a logical and mathematical, but also an epistemic incompleteness of mathematics*: No consistent mathematical theory *can justify all of mathematics*. *The incompleteness of mathematics poses the issue of justifying mathematical cornerstones*. Examples of mathematical cornerstones are *principles expressing fundamental properties* of the most basic mathematical concepts or principles expressing the *reliability of our best mathematical theories*.

There has been a shift in epistemology towards the idea that *mathematical cornerstones, or basic mathematical axioms and concepts, can only be entitled*. Crucially, entitlement behaves quite differently in contrast to more traditional mathematical justifications: Entitlement is a default type of justification, which is never the result of some cognitive, evidential work. Importantly, entitlements can be defeated or undercut. Entitlement drastically changes the traditional epistemological landscape and, if successfully integrated into our epistemology of mathematics, it has the potential to provide a more complete picture of mathematical practices and to provide a more fine-grained conceptual framework for the epistemology of mathematics. Yet, the notion of mathematical entitlement must still be fully developed. Moreover, no attempt has been made towards a systematic integration of entitlement into the standard proof-based epistemology. *Little is known about the normative force and epistemic role of entitlement*. Moreover, we know little about the *impact of entitlement for the question of the objectivity of mathematics*.

The project's main goal is to make substantial progress towards a *better understanding of entitlement and its integration* in our proof-based epistemology. The project thrives towards its aim by pursuing three objectives:

- (1) Determine the normative force of entitlement as justification
- (2) Evaluate the epistemic role of entitlement for mathematical justification
- (3) Analyse the impact of entitlement for the objectivity of mathematics

Research Methodology

The proposed project is unprecedented, and its novelty lies both in its expected results and methodology: The project's methodology essentially departs from the current approaches to the epistemology and philosophy of mathematics by expanding our set of tools from (philosophical and mathematical) logic and concepts traditionally used in philosophy of mathematics with genuine methods and concepts from general and formal epistemology. This project will actively engage, analyse and employ these concepts. The project departs from the standard approach to the epistemology of mathematics also from a more *programmatically perspective*: In an important sense, the project *wants to reinforce the authority of epistemology in epistemological debates in the philosophy of mathematics*, against the trend that mathematics might have no epistemological tribunal. Crucially, the project recognizes the *partial authority of mathematics* and the crucial importance of mathematical and formal work for philosophy and epistemology, but wants to give epistemology its authority for the relevant philosophical questions.

Expected Impact

The results of the project are expected to have a direct scientific impact on fields and disciplines within philosophy, including philosophy and foundations of mathematics, epistemology, philosophy of mathematical practice, and philosophy of science. The integration of entitlement in our epistemology through the construction of a flexible and generalisable framework promises to have a lasting impact well beyond the time of the project. Finally, the project's unprecedented methodology will set a foundation for future research, thereby having an impact on the scientific practice beyond the time of the project by enabling future dialogue and collaboration between traditional philosophers of mathematics and of science, formal philosophers, and epistemologists.