The main goal of the research project is to obtain new results and answer a number of questions raised in the literature concerning ideals on the set of natural numbers, and to apply the obtained results in studies of sets of limit points. The most basic example of an ideal on the set of natural numbers is the collection of all finite subsets of natural numbers. Other examples include, for instance, the collection of sets of asymptotic density zero. Ideals on the set of natural numbers are one more tool – standard in set theory and topology – to study the real line.

In the first part of the proposed project we shall investigate Borel complexity of ideals on natural numbers. This is a natural way of describing, how complicated is a given ideal. It is known that there are no $\mathtt{G}_\delta$ ideals. Moreover, $\mathtt{F}_\sigma$ ideals have a nice combinatorial characterization due to Mazur. In the case of the next level of complexity – $\mathtt{F}_{\sigma\delta}$ ideals – we only have a combinatorial characterization (due to Solecki) for the class of so-called P-ideals. Farah conjectured that its slight modification should work for all $\mathtt{F}_{\sigma\delta}$ ideals. In the project we want to investigate this conjecture.

The above problems are related to studies of sets of limit points. For a given ideal $\mathcal{I}$ on the natural numbers and sequence $x = (x_n)_{n\in\mathbb{N}}$ in a topological space, the set of limit points consists of all points which are limits of some subsequences $x = (x_n)_{n\in A}$ of x with the index set A not belonging to the ideal $\mathcal{I}$. In the second part of the project, we investigate such sets and their modifications obtained by replacing ordinary convergence with IP-convergence (connected to Hindman's finite sums theorem) or R-convergence (connected to Ramsey's theorem). Those two notions of convergence have been introduced by Furstenberg and Weiss (IP-convergence) and by Bergelson, Zelada, Bojańczyk, Kopczyński and Toruńczyk (R-convergence) in the context of ergodic theory.