For many years, problems related to representations of natural numbers as sums of positive integers like Goldbach's conjecture, Pollock's conjectures or Waring's problem have been playing the crucial role in Number Theory. In fact, that kind of problems might be reformulated in terms of both partition and composition theories.

Formally, for a fixed multiset $A$ of positive integers an $A$-partition of a natural number $n$ is such a sequence $\lambda = (\lambda_1, \ldots, \lambda_j)$ of elements from $A$ that

$$n = \lambda_1 + \cdots + \lambda_j.$$

Moreover, two $A$-partitions are considered the same if they differ only in the order of their parts. That one property distinguishes $A$-partitions from $A$-compositions, where the order of parts plays the role. The total number of $A$-partitions and $A$-compositions of a given parameter $n$ are denoted by $p_A(n)$ and $c_A(n)$, respectively. In particular, if $A = \mathbb{N}_+$, then we obtain the (classical) partition function $p(n) := p_{\mathbb{N}_+}(n)$ and the composition function $c(n) := c_{\mathbb{N}_+}(n)$.

In general, research devoted to both $A$-partitions and $A$-compositions are complicated but attractive. For the last decade, logarithmic properties (log-properties) like logarithmic concavity (log-concavity) and logarithmic convexity (log-convexity), logarithmic Fibonacci property (log-Fibonacci) or logarithmic subadditivity (log-subadditivity) and logarithmic superadditivity (log-superadditivity) have been attracting the special attention of many mathematicians. Let us recall that a sequence $(a_n)_{n=1}^{\infty} \in \mathbb{R}^{\infty}$ is log-concave (log-convex) if the inequality $a_n^2 > a_{n-1}a_{n+1}$ ($a_n^2 < a_{n-1}a_{n+1}$) is satisfied for all but finitely many numbers $n$. We say that $(a_n)_{n=1}^{\infty} \in \mathbb{R}^{\infty}$ is log-Fibonacci if there exists $\varepsilon \in \{-1, 1\}$ such that the equality $\operatorname{sgn}(a_n^2 - a_{n-1}a_{n+1}) = (-1)^n\varepsilon$ holds for all but finitely many numbers $n$. Moreover, a sequence $(a_n)_{n=1}^{\infty}$ is log-subadditive if the inequality $a_m a_n > a_{m+n}$ is valid for all large values of parameters $m$ and $n$. It is log-superadditive if the converse inequality is true for all but finitely numbers $m$ and $n$.

It is worth pointing out at this point that a lot of combinatorial sequences satisfying the aforementioned log-properties have various application in practice, for instance, in the so-called white noise theory, quantum physics and mathematical biology.

The main aim of the project is to significantly extend the current knowledge related to log-properties of both functions $p_A(n)$ and $c_A(n)$, and some of their generalizations. In the last few years, I obtained several results of that kind for all $A$-partition functions with a finite multiset $A$ (i.e. $\#A < \infty$). Therefore, it seems to be natural to extend the received properties and investigate their analogues for corresponding $A$-composition functions for finite $A$. The case of $\#A = \infty$ is more complicated. Until now, there have been only discovered partial results for some specific choices of multisets $A$ and the associated functions $p_A(n)$ and $c_A(n)$. Since the main difficulty of that case is the lack of asymptotic estimates for the considered $A$-partition and $A$-composition functions, it is more convenient and reasonable to try to deal with their log-properties by either using a combinatorial approach or adding some restrictions on the structure of $A$. The other efficient method to investigate the subject is to generalize the functions $p_A(n)$ and $c_A(n)$ by following Gian-Carlo Rota's advice, and pass from the discrete objects to the continuous ones by considering some special families of polynomials.