DESCRIPTION FOR THE GENERAL PUBLIC

Graphs are abstract structures whose role is to model interactions between pairs of objects. Such objects are represented by the *vertices* of the graph and the interactions are represented by the *edges* of the graph. An important family of graphs are *geometric intersection graphs*, in which vertices correspond to geometric objects, such as intervals on the real line, arcs of a circle, chords of a circle, or disks or continuous curves in the plane, and edges to the pairs of intersecting objects. By considering objects of different kind, we obtain different families of graphs; for example, *interval graphs* are the intersection graphs of intervals on the real line, *circular arc graphs* and *circle graphs* are the intersection graphs of arcs and chords of a circle, respectively. Graphs of this kind are widely used to model various real problems, and efficient algorithms working on such graphs find practical applications in areas like resource allocation or VLSI design.

Intersection graphs of simple geometric objects constitute important benchmarks for many considerations of algorithmic graph theory. A number of computational problems, which are proved to be computationally hard in the class of all graphs, in some classes of geometric intersection graphs can be solved efficiently by polynomial-time algorithms. Similarly, a number of structural problems, still open in the class of all graphs, have found a solution in some geometric intersection graph classes. The study of these graph classes over the years resulted in the development of many algorithmic methods and proof techniques, that have found applications to broader classes of graphs, including the class of all graphs. Still, some important problems of graph theory (e.g. Hadwiger's conjecture) are not only open in the class of all graphs, but also in some classes of intersection graphs of simple geometric objects. So, graphs from these classes, despite their definitional simplicity, turn out to have quite complicated and intriguing structure.

The central algorithmic problem for a graph class $\mathscr{G}$ is to *recognize* graphs from this class, that is, to test whether a graph given in the input belongs to the class $\mathscr{G}$. For many geometric intersection graph classes the recognition problem has found a satisfactory solution – either a polynomial-time algorithm or a proof of its computational hardness. Recognition problems can be generalized in many ways, and two of them, *certifying recognition* and *graph modification* problems, will be the subject of the research in this project. *Certifying recognition algorithms* test whether a given graph belongs to the class $\mathscr{G}$ and for each answer they provide a certificate of its correctness. A *certificate* is an easily verifiable evidence confirming that the response has not been corrupted by a bug in the implementation of the algorithm. *Graph modification algorithms*, in turn, test whether a given graph, typically not from $\mathscr{G}$, can be slightly modified so that it becomes a graph from $\mathscr{G}$. Modification problems find practical applications to correcting errors in experimental data.

A fundamental structural problem related to a fixed graph class is their characterization in terms of *forbidden structures*. For a graph class $\mathscr{G}$ we want to find a family of structures $\mathscr{F}$ such that the following property holds: a graph belongs to $\mathscr{G}$ if and only if it does not contain a structure from the family $\mathscr{F}$. Such a characterization is particularly useful in the construction of certifying and graph modification algorithms for geometric intersection graph classes. Certifying recognition algorithms check whether a given graph belongs to class $\mathscr{G}$; in the case of a positive answer, they provide a model of the graph appropriate for the class $\mathscr{G}$, in the case of a negative answer, they provide a certificate in the form of a forbidden structure for the class $\mathscr{G}$ found in the input graph. Graph modification algorithms, in turn, check whether all forbidden structures can be removed from a graph by slightly modifying the structure of the graph. So, certifying and graph modification algorithms are in some sense complementary to each other and their construction usually requires a neat characterization of the graph class in terms of forbidden structures. For many important geometric intersection graph classes (interval graphs, chordal graphs, permutation graphs) their characterization in terms of *minimal forbidden graphs* for these graph classes is known and for some of them efficient certifying and graph modification algorithms have been designed. Such a characterization is not known for the class of circular arc graphs and the class of circle graphs. Obtaining such a characterization for these graph classes and designing certifying and graph modification algorithms for graph classes in which these problems are still open is one of the main goals of this project.

We plan to use the obtained structural results on geometric intersection graphs to solve certain problems related to the *graph coloring* problem. One of the most important questions in this area is: what structures are forced in a graph if we need to use at least k color to properly color the graph (color the vertices of the graph such that any two adjacent must obtain distinct colors)? The answer to this question is postulated by Hadwiger's conjecture, which precisely describes structures that should be enforced (these are so-called *minors of k-clique*). Hadwiger's conjecture is widely considered to be one of the most important open problems in graph theory. For some cases it is is equivalent to the *four color theorem*, which states that every planar graph is 4-colourable (the proof of which relies on machine-based computations). The research on Hadwiger's conjecture in the classes of intersection graphs, especially in the class of circular arc graphs and circle graphs, may shed a new light on this conjecture. In particular, it might answer the question whether there are some other reasons, different than the four color theorem, which make this conjecture so difficult to prove.