Let us consider two unit spheres, respectively n-dimensional and k-dimensional (that is, sets of points at a distance of 1 from the origin in $\mathbb{R}^{n+1}$ and $\mathbb{R}^{k+1}$). A long-studied question in topology is:

Question 1. *In how many ways can we "wrap" the n-dimensional sphere around the k-dimensional sphere?*

It turns out that the number of such ways can be quantified using the homotopy group. This is an algebraic object denoted by the symbol $\pi_n(\mathbb{S}^k)$. For example, when $n = k = 1$, we have $\pi_1(\mathbb{S}^1) = \mathbb{Z}$, which corresponds to the intuition that a circle can be wrapped around another circle in infinitely many ways. Things get more interesting when $n > k$. For instance, for $n = 4$ and $k = 3$, we have $\pi_4(\mathbb{S}^3) = \mathbb{Z}/2$, which means that the four-dimensional sphere can be wrapped around the three-dimensional sphere in exactly one significant way. In general, for higher values of n and k, the homotopy groups of spheres are very difficult to compute, and even harder to intuitively understand.

This fact inspires the following question:

Question 2. *Given a fixed way of wrapping $\mathbb{S}^n$ around $\mathbb{S}^k$, can it be represented in some particularly simple way?*

For example, double wrapping $\mathbb{S}^1$ around $\mathbb{S}^1$ can be represented by the equation $(x, y) \mapsto (x^2 - y^2, 2xy)$. In 1987, Bochnak and Kucharz posed the following version of Question 2:

Question 3. *Can every way of wrapping $\mathbb{S}^n$ around $\mathbb{S}^k$ be represented by a regular map, i.e., a rational map with a non-vanishing denominator?*

Everything indicates that the answer to the above question is positive. So far, it has been answered when $n \leq k + 5$ and when $k = 1, 2, 3, 4$, or 7. Despite this, we still lack a method to determine whether the answer is positive in full generality.

While studying regular maps between spheres, a certain strengthening of Question 3 naturally arises, in which the regular map is replaced by a certain narrower class of maps called *strongly regular* maps. This is a completely new idea that could have a huge impact on our understanding of the phenomena behind the current positive results. The question which homotopy classes of maps between spheres, as well as between other real manifolds, are represented by strongly regular maps is interesting in itself and can provide much information in the context of Question 3. This is the focus of the current project.

A particularly important and relatively achievable task in the project is to study strongly regular maps from spheres to the orthogonal groups $\mathrm{O}(n)$ and unitary groups $\mathrm{U}(n)$. Due to a certain well-known construction, these maps induce certain results for spheres via the so-called J-homomorphism, which are of great significance for Question 3.