

Popular description of the project

The project is devoted to stochastic control problems which arise in a natural way in various applications. The first topic is to study **time inconsistent control problems**. These are problems for which famous Bellman optimality principle does not work. This principle says that optimal control for a functional in given time interval and is also optimal for time subintervals. There are many problems for which this is not the case. Typical situation is when we have general discount rate with the property that discounting for times interval $t+s$ gives us more than product of discounting for time t and for time s separately. In such cases we have time dependent control problem. We can solve it increasing dimension of the model (which creates important computational problems), or consider the problem from economical point of view looking for so called equilibria among strategies consisting of the functions of the current value of the state process. In the project the second approach will be considered mainly for various controls and reward functionals. We shall also identify long time horizon problems for which time inconsistent control problems can be solved using control functions which are optimal for an auxiliary time consistent control problem.

The second topic concerns **asymptotic behavior and stability of controlled models**. We are interested to know how certain models behave for long time horizon (when time converges to infinity), or how reward functionals depend on constants and controls. Proper model should be robust which means that small changes of time, parameters and controls should have neglected impact. Another asymptotics we are interested is when we have a number of agents with controlled financial positions depending on the empirical measure of positions of all agents, and we want to know what kind of limit we obtain when number of agent goes to infinity. It is worth to point out that we consider time or number of agents going to infinity, when we don't know exact time horizon and number of agents but we are aware that both values are large.

The last topic of the project concerns mathematics of finance. We study financial markets considering frictions, and related econometric and statistical challenges. Namely we consider financial markets with transactions costs: proportional, concave, convex with respect to the volume of transaction. We also assume additional fixed costs. In the case of small investor for larger purchases (or sales) the broker can diminish (increase) the price. The situation is also typical for real estate markets or currency markets. On the other hand large investor has an impact on the market increasing the price when he is buying and decreasing it when he is selling. Consequently we have concave or convex transaction cost. Fixed cost is related to the broker's fee. We want to study problems of arbitrage on such markets (making profit without risk) as well as convergence of value functionals and optimal investment strategies when fixed cost converges to zero. Finally we would like to consider risk sensitive portfolio optimization with general noise, which because of the form of the functional measures not only expected value but also variance of the reward.