

# Meta-theoretic and inter-theoretic reductions in the foundations of mathematics

In the foundations of mathematics, various reducibility relations are often analyzed. In the current project, we investigate two broad categories of such reductions: meta-theoretic reductions and inter-theoretic reductions. Meta-theoretic reductions can be thought of as providing formalizations of certain intuitive notions (such as the notion of a natural number, a set, etc.). They are reductions of informal concepts to formal theories. Inter-theoretic reductions reduce one theory to another formal theory. Intuitively, such reductions of one theory to another show that, in some sense, the reducing theory can mimic the reduced theory. There are many natural examples of such reductions. In the project, we want to investigate their epistemic status from the perspective of the conceptual content of the theory.

Many natural formal theories occurring in the foundations of mathematics seem to have a designated subject matter. For example, arithmetical theories are often described as first-order theories of the arithmetic of natural numbers; certain subsystems of second-order arithmetic are designed to grasp the real number analysis; axiomatic theories of truth are meant to formalize the notion of truth used in mathematical and informal reasoning; theories of sets are formal theories grasping the informal concept of set. Meta-theoretic reductions state that certain concepts or foundational standpoints are reducible to a formal theory. They are crucial for formal investigations into the foundations of mathematics: without them, there seems to be no connection between a formal theory and a (broadly construed) subject matter of an area of mathematics. They also provide a good sense of the conceptual content of a theory—theories formalizing an area of mathematics or a foundational standpoint are intuitively about the concepts used in the area of mathematics or the foundational standpoint. Our project aims at analyzing this intuitive idea philosophically and making it formally precise.

Inter-theoretical reductions are crucial tools in logical analysis of formal theories. What philosophical consequences one can draw from the existence of a given reducibility relation between two formal theories? Our main objective in this part of the project is to analyze this question in the context of the conceptual content of a theory. How strong a reducibility relation do we need to preserve the conceptual content between theories? Also, it seems that in acceptance of a formal theory formalizing some concept (e.g. accepting ZFC as formalizing a concept of set), one seems to be implicitly committed to acceptance of concepts that are not explicitly formalized by the theory (e.g. a concept of a natural number). We can therefore ask, how strong a reducibility relation between a theory formalizing one concept and a theory formalizing a different concept should be to give rise to such conceptual commitments. Presumably, the target reducibility relation is rather strong. Hence, one can ask about the epistemic interpretation of weaker kinds of reductions between theories. In the project, we propose an interpretation of certain natural reducibility relations as a good measure of conceptual distance between theories.