

## LOW DEGREE COVERINGS OF GENUS 2 CURVES

The main object of our studies are coverings of smooth curves defined over complex numbers. In the real world such curves are, in fact, 2-dimensional surfaces. To realise a covering one can imagine two copies of a plane lying above the same plane. This is an example of an unramified double covering. As shapes of the underlying objects may vary, the geometry of coverings can become much more sophisticated.

To every such covering one can associate a special variety, called the Prym variety of the cover. The operation of taking the Prym variety defines a map from the space of all coverings of certain type (up to isomorphism) to the space of abelian varieties of certain dimension and polarisation type. This map provides a bridge between the moduli space of curves and the moduli space of abelian varieties. It is an important example of a general philosophy in mathematics when one can study an object via another object using some kind of a link between them. As both curves and abelian varieties are very important in algebraic geometry, this map has been studied for more than a century.

The focus of mathematicians of the 20th century in this area was on the Prym map of double coverings, and only recently other coverings started to gain attention in the community. However, the Prym map of cyclic coverings of composite (i.e. non-prime, such as 4 and 6) order has not been studied yet. This project is the first attempt in this direction.

In the project we always assume that the curve we cover has genus 2. From the point of view of topology, every such curve is isomorphic to a surface of a "donut with two holes". The main motivation to consider genus 2 curves comes from the fact that such curves are double coverings of a complex projective plane. This gives an additional geometric structure on the Prym variety. The coverings we want to consider are cyclic unramified coverings of degrees 4 and 6. We choose these degrees of coverings because Prym varieties of such covers decompose into subvarieties of relatively low dimension (more precisely, curves and surfaces). Thus, it is a natural first step in this direction since much more is known for curves and surfaces than for higher dimensional varieties.

The first goal of the project is to understand the decomposition of the Prym varieties into smaller pieces which comes from the symmetries of the coverings. Using this decomposition we would like to understand what distinguishes such Prym varieties in the space of all abelian varieties of a given dimension and polarisation type. In this way we can define a "refined" Prym map and study its fibers (i.e. sets of coverings which give the same Prym variety) which for a new map are always non-empty. Dimension count in the case of covers of degree 4 (resp. 6) shows that a general fiber is a curve (resp. a finite set of points). Therefore, the goal in the former case is to understand what kinds of curves we can get as fibers of the Prym map while in the latter case the goal is to compute how many coverings are in a general fiber.

In order to make the results of the project accessible to more researchers we plan to determine equations of intermediate curves and period matrices of Prym varieties and their components.