APPLICATION OF STOCHASTIC ANALYSIS IN DETERMINING SCALING LIMITS IN SOME MODELS IN MATHEMATICAL PHYSICS

(a brief general description)

Our research proposal concerns some mathematical problems that stem from the passage from microscopic to macroscopic description of certain physical systems. More precisely, there are three particular examples that we have in mind. First, concerns the rigorous derivation of macroscopic laws that govern the behavior of thermodynamical systems by taking the limit from a microscopic model under a suitable space-time scaling. In particular we wish to understand this transition for a chain of interacting oscillators, which is a classical model of heat propagation in statistical mechanics. Our goal is to find the description of the evolution of macroscopic quantities such as temperature, pressure and density from their microscopic counterparts (microscopic energy, momentum and stretch) in stochastically perturbed muti-dimensional harmonic crystals as well as in some simple chains of interacting anharmonic oscillators, where the anharmonicity concerns only its pinning potential. A topic of a particular interest is the emergence of boundary conditions at the macroscopic scales for an open system, i.e. a system that is in contact with an external environment, e.g. via heat baths, or by an action of some outside force. We would like to understand, on a mathematical level, how the work done by an external force is transformed into the heat.

The second problem concerns the nature of fluctuations of the solution of the Kardar-Parisi-Zhang (KPZ) equation and a closely related to it problem of fluctuations of the endpoint of a directed random polymer. The KPZ equation is a default model for random surface growth in statistical mechanics. The study of the equation in one spatial dimension and with a space-time white noise has witnessed important progress in recent years. In particular, for the KPZ equation on the whole real line, it was shown in that, under the so called $1 : 2 : 3$ scaling, the centered solution, also referred to as a height function, converges to a limiting object that belongs to the Kardar-Parisi-Zhang (KPZ) universality class. In the series of our papers, we have studied the fluctuations of the solutions of the KPZ equation posed on a torus of a fixed size. We have shown then that these fluctuations are Gaussian for a torus. If the size of the torus grows in time one can observe the passage from the Gaussian fluctuations to the non-Gaussian ones that characterizes the KPZ universality class. We plan to investigate closely this problem in the present project. In addition, in this part we are also going to study a closely related topic of fluctuations of the path of a directed random polymer on a cylinder.

Finally, in the third part of our proposal we consider a model of transport in a turbulent flow - the so called *passive tracer model*. It is one of the most popular models of transport in a complicated medium considered in statistical hydrodynamics. Its formulation goes back to the 1920-s and the investigation of turbulent diffusion phenomenon by G. I. Taylor. In oceanography it serves as a model of turbulent transport of substance by ocean currents, provides the description of mixing of nutrients in the ocean. It is also used in geophysical and environmental problems to describe the advection and dispersion of pollutants in the atmosphere, chemical reactions in industrial flows. In the aforementioned model the motion of a tracer particle is described by a diffusion with a drift given by a random vector field. The basic problem is to determine the statistical properties of the tracer from the known properties of the random field. A particular question of interest is the validity of the central limit theorem for the tracer. The case when the drift is incompressible is by now quite well understood. We plan to investigate the validity of the central limit theorem for diffusions with a Gaussian and *compressible* drift.