Extremes of Growth-Fragmentations and related processes

Fragmentation is a process where an object breaks into smaller fragments over time. A natural example is the fragmentation of the earth's crust during an earthquake. These events occur randomly and cause the surface to fracture into variously sized and shaped segments. Fragmentation is also observed in other fields: DNA fragmentation in biology, atom fission in nuclear physics, hard drive fragmentation in computer science, and many others.

Figure 1 presents a simple geometric example of fragmentation. It shows the fragmentation of a rectangle at discrete time intervals. At each step, the smallest rectangles split either along their base or height into two smaller rectangles.

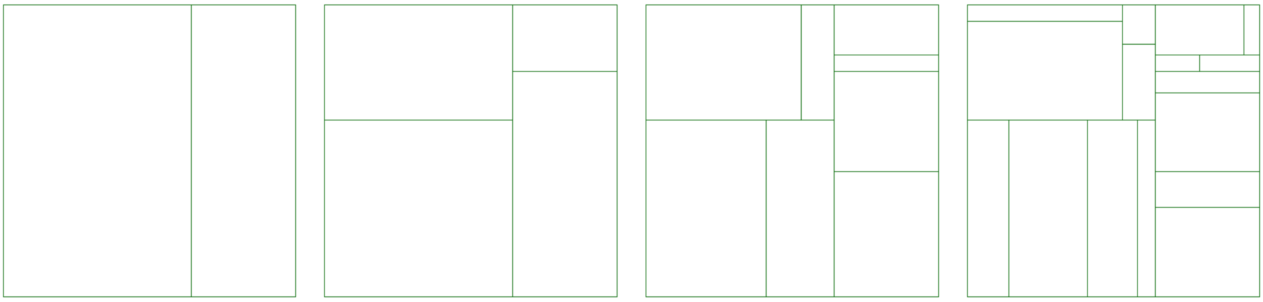

Figure 1: Example of fragmentation

Classical fragmentation processes impose strict rules on the properties of fragmentations. Such as that the size of fragment between its splits must be constant or that the time of split can only depend on fragment size. These rules do not always apply in nature. For instance, between earthquakes, changes in land size can occur due to erosion or human activities. Additionally, land with a more longitudinal shape is more prone to breaking during earthquakes. Therefore, the main aim of this project is to study generalisations of classical fragmentation processes that allow for these variations and dependencies, providing a more accurate representation of natural fragmentation phenomena.

We propose to investigate two more recently introduced models. The first is the *growth-fragmentation process*, which allows fragments to randomly grow or shrink over time. The second model is the *spatially-dependent fragmentation process*. Similar to the earlier example, rectangles break into progressively smaller rectangles, but in this model, the splitting times are arbitrary and the rate of splitting depends on the shape of the fragment. Long, thin rectangles are more likely to break quickly and along their longest side.

We aim to investigate the asymptotic behaviour over time of the sizes of the largest and smallest fragments in these two models, referred to as extreme behaviour.

Classical fragmentation processes can be interpreted using *branching processes*. For us, a branching process refers to a evolving in time system of particles that move and at a random time die and split into a random number of offspring particles. Similar relationships with branching processes exist for the models we are studying, so part of the project will be formulated in terms of those processes.

The problems posed in the project, especially concerning the spatially-dependent fragmentation process, will require new and innovative ideas, which will be integrated with more traditional methods of studying the extreme behavior of branching processes, such as *spine decomposition* and the *many-to-one formula*.