

## FAIR DIVISION OF INDIVISIBLE GOODS UNDER INCOMPLETE INFORMATION

In our project we focus on the fair division of indivisible goods in the presence of incomplete information. Given a set of goods and a set of agents having preferences defined on these goods, the goal is to assign the goods to agents so that some fairness criteria will be satisfied. We assume that the preferences of agents are represented by the so-called valuation functions, corresponding with the subjective evaluation of the value of the goods.

There are various fairness criteria that can be considered. The most commonly used are envy-freeness (EF), proportionality (PROP), equitability (EQ) and Pareto optimality (PO). EF means that after the allocation of goods no agent envies another one (i.e., everyone is convinced that their own bundle of goods is at least as good as the bundle of anyone else with respect to their valuation function). In the case of PROP everyone is convinced that they received at least a proportional share of all the goods. EQ assumes that everyone is convinced that they received exactly the same share (with respect to their valuation function). PO means that there is no other allocation such that everyone would do no worse than in the current one and at least one agent strictly better.

All the mentioned criteria are well analyzed and the allocations that fulfill them can be found in the case of infinitely divisible goods. However, in the case of indivisible goods some of them do not need to exist. For that reason some relaxations are considered, in particular envy-freeness up to any good (EFX), where for every pair of agents  $i, j$  and for every good  $g$ , even if agent  $i$  envies agent  $j$ , this envy disappears if  $j$  gets rid of  $g$ , and maximin share fairness (MMS), where each agent receives at least as much (in terms of their valuation function) as the maximum over all possible allocations of the value of the least valued member of the allocation. Although MMS is a significant relaxation of PROP, it does not need to exist, while it is not known whether EFX always exists or not. For that reason the approximation variants of both criteria are also considered.

The incompleteness of information present in the real world motivated us to consider the local variants of the problem. Assume that we consider a fair division problem in a social network (or knowledge graph). Now, any agent compares the obtained bundle of goods not with all the other agents, but only with those visible in the network. We are going to define the local versions of the fairness criteria in a natural way and analyze their properties. In particular, we are interested in the following two measures: local envy-freeness up to any good (LEFX), and local maximin share fairness (LMMS), the analogue of MMS. We also plan to consider the approximation versions of these fairness criteria.

We want to identify the structures of knowledge graphs that guarantee the existence of locally fair division. Since part of the proofs will be constructive, we will also present some fair division algorithms and analyze their properties.

Finally, we want to investigate the relations between the local and global fairness. It is easy to observe that some locally fair allocations can be globally unfair. Our goal is to investigate, how far can a locally fair allocation be from the global fairness and how is this distance related to the structure of the knowledge graph. In the future this could be the base for the models allowing to explain how the structure of the social connections can influence the level of inequalities in the society. Our preliminary research shows that not only obvious graph metrics, like diameter or average degree, are important in this context, but also some other parameters like chromatic number.

In our work we will use mostly the mathematical methods typical for Econometrics, Mathematical Economics and Operations Research, like deduction, mathematical induction and matrix algebra. In the case of non-constructive proofs, we plan to use more advanced tools: the Probabilistic Method, polynomial algebra and duality theorems, including KKT (after relaxing the indivisibility condition). The results of the simulation experiments will be analyzed using standard statistical methods.