Relation based semantics: axiomatic and tableaux proof-calculi

One of the most important aspects of human intelligence is our ability to reason: that is, to synthesise new conclusions from given information or assumptions. But how do we separate the wheat from the chaff? Are the reasoning steps linking the assumptions to the conclusion valid? Will this line of reasoning be accepted by others? Could the conclusion be different under some other form of reasoning? Questions such as these have led philosophers to postulate that our reasoning is based upon systems of formal rules of inference capturing different styles of reasoning.

The scientific discipline that deals with providing methods for validating reasoning is called Logic. Modern Logic also provides criteria for judging the correctness of reasoning by stipulating a finite collection of rules of inference which connect our assumptions to our conclusion. Depending on the type of reasoning and our motivation, different logics arise by stipulating different rules of inference and different styles of proof. For examples, one of the most fundamental rules of inference is called Modus Ponens: if A is true, and, if A implies B is true, then, B is true. It captures the common linguistic structure of the assumptions and the conclusion, regardless of the actual values of A and B.

We now understand the logic of reasoning using linguistic expressions built from the connectives "and", "or", "not" and "implies" using inference rules such as Modus Ponens. But our understanding of more complicated philosophical notions such as "necessary", "possibly", "knows", "believes", "permitted" and "obliged" are still not clear. Philosophers have therefore invented many non-classical logics utilising rules of inference that apply to such modal connectives. Unlike the classical reasoning described above by the Modus Ponens rule, these principles do not always capture simple structural properties of the assumptions. Instead, they typically rely on the modal relationships that hold between the simpler sentences A and B. Is B caused by A, or is B contained in A, for example.

Logics determined with relation based semantics are a promising new type of logic whose most basic principle is that two sentences A and B can be related by some relationship $R(A, B)$ without specifying its actual value as causation or containment. Thus these logics allow us to capture many non-classical notions in one fell swoop.

The aim of the project is to develop methods to check the correctness of reasoning for logics determined with relation based semantics using collections of inference rules called axiomatic calculi and tableaux calculi. Axiomatic calculi use forward proof to infer conclusions from the assumptions. Tableaux calculi use backward proof to reduce the conclusion to the assumptions. Axiomatic calculi are modular, switching from one logic to another by simply adding or deleting axioms, but finding forward proofs is difficult as we have no way to guide the inference rules towards the conclusion. Traditional tableaux calculi are not modular but can be extended by extra machinery to make them so. Their biggest advantage is that they are "goal-directed" because we can backchain from the conclusion towards our known assumptions. Both are vital for a deeper understanding of relation based semantics. The project therefore aims to solve the following problems for relation based semantics:

- develop the philosophical foundations of relation based semantics;
- determine when R can be expressed by adding new object level connectives;
- find axioms corresponding to different properties of R when it is so expressible;
- find methods for dealing with relations R which are not so expressible;
- find the limits of traditional tableaux calculi for relation based semantics;
- develop extended tableaux calculi in which the relation can be expressed directly.