Tensor rank and applications to signature tensors of paths

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Tensors are multi-dimensional tables filled with numbers. Even if they may look like simple objects, they are useful in many different scientific areas. They can record the result of a chemical experiment or the complexity of an algorithm. Statisticians use tensors (called moments and cumulants) to study random variables. The list of applications of tensors is very long.

The rank of a tensor is a number that measures how much information is encoded in that tensor. In this project I study the rank of tensors from the point of view of algebraic geometry. For every natural number r, we can consider the variety parametrizing tensors of rank at most r. It is in general very difficult to compute its geometric properties, such as its dimension. There is a conjecture that gives us a possible formula for the dimension of these varieties. I plan to prove that this conjecture is true for an important class of tensors, called partially symmetric.

The second part of my project is about a concrete application of tensors. Given a path X, it is possible to associate to X a sequence of tensors that encode the properties of X. These tensors are called the signatures of X. Recently we proved that if the path is very simple - namely, if it is a straight line segment - then its signatures are very simple - namely, they all have rank 1. The second goal of this project is to generalize this result, and prove that more complicated classes of paths, like polynomial of piecewise linear paths, can also be characterized by conditions on the rank of their signatures.