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Nonlinear systems – new challenges

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Our research aims to meet new challenges in systems of nonlinear partial differential equations. Many fascinating physical or biological phenomena involve the deeply nontrivial mathematical description. The theory of the Magnetohydrodynamic Dynamo describing the existence of a magnetic field of stellar objects, including the Earth, is one of the phenomena that we will study in this project. These studies are related to a complex system of hydrodynamics that takes into account the magnetic field, thermal effects, convection phenomenon and compressibility.

Our research program also includes widely understood nonlocal processes, which are described by the appearance of integral terms in the equations of hydrodynamics. This type of description has been proposed for modeling of collective behavior in the social sciences and life sciences, and allows for the presentation of a rich variety of behaviors depending on the proposed potentials in integral terms. We will direct our attention also to nonlocal terms in the context of innovative traffic models.

It is worth noting that indeed a wide spectrum of problems can be described using the equations of hydrodynamics. This description is possible on different scales level. The most obvious distinction will be the micro-, meso- and macroscopic level. Equally interesting, however, is the asymptotic analysis of certain parameters, such as high pressure, low Mach number or low viscosity. Relationships between models taking into account various limit passages are an important research goal that allows to verify the equations.

The scientific community has come to terms with the fact that the existence of classical solutions that are global in time for fluid mechanics systems cannot be expected. That is why the methods of functional analysis have developed so widely in order to search for weak solutions, and sometimes even solutions in a weaker sense – measure-valued solutions. However, the mere existence of solutions is not fully satisfactory. One asks further questions on properties of solutions, like uniqueness. We will pay particular attention towards the relative entropy method. In essence, it measures the distance between two solutions of a given system in comparison to the distance of their initial data. The method of relative entropy, which we will consider, not necessarily always provides uniqueness, but proved to have an exceedingly diverse range of applications. We will use it to show the property of a conditional uniqueness stability numerical schemes, as well as limit passages between different thermomechanical theories.

Weak convergence methods are basic tools related to systems of nonlinear partial differential equations. In the description of physical, biological phenomena and social processes, with few exceptions, nonlinearities and/or nonlocal terms are inevitable. Hence, the development of these methods is a definite priority. A rigorous analysis of the model is the starting point for building correct numerical schemes and computer simulations, and at the end of this process to draw conclusions helpful in predicting and controlling various type of phenomena. Accurate prediction of processes gives the possibility of avoiding expensive, or sometimes even impossible to perform, experiments.