Topological, geometric and ergodic problems of complex dynamics

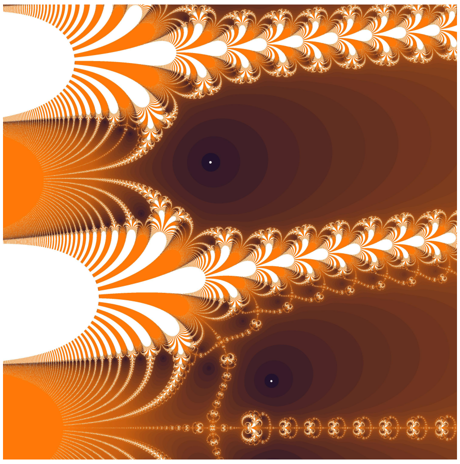
Abstract for the general public

The project presented here concerns questions of complex dynamics, which is a part of the theory of *dynamic systems*. In this theory, one studies the evolution of a given space (set) X over a long period of time. In our case, the passage of time is modelled in a *discrete* (stepwise) manner, and the change of the system at the transition from a given moment to the next one is described by a transformation (function) $f\colon X \to X$. In the *autonomous* case, the function f (evolution rule) does not change over time. In this case the change of the system after time n, where n is a natural number, is described by the n-th *iteration* (multiple composition)

$$f^n = f \circ \cdots \circ f \quad (n \text{ times}).$$

Dynamical systems theory, developed in the 20th century, has now become an important part of modern mathematics, providing research tools used in other fields, and having numerous applications in the natural and social sciences.

The project presented here concerns the theory of iteration of holomorphic maps of a complex variable. In this case, the space X is a complex plane or the Riemann sphere, while the transformation f is a rational, entire or meromorphic function. The space X can be then divided into two dynamically defined subsets – *the Fatou set*, where the iterations of the transformation behave in a regular way, and *the Julia set*, where the dynamics is chaotic. It turns out that the Julia set is has typically a complicated topological and geometric structure (see figure below).



A significant part of the project concerns the *transcendental* case, when the point at infinity is an essential singularity of the function f. In this situation, the Julia set may contain invariant sets of complex structure, for instance the so-called *Cantor bouquets* or Knaster-type *indecomposable continua*. Within the project, we would like to study the topological and geometrical properties of such sets. We will also consider the structure of the boundaries of connected components of the Fatou set and determine various types of their dimensions. It is worth mentioning here that these sets often exhibit *self-similarity*, as well as other fractal properties.

Another part of the project is devoted to *non-autonomous* and *random* holomorphic dynamics. Here we allow for the situation where the rule describing the evolution of a system can change over time. Then, instead of one transformation f describing this evolution, we have a choice of many different transformations. In particular, we may choose this transformation each time in a random way. As a part of the project, we will study, among other questions, what effect the introduction of such randomness has on the structure and properties of the Julia set.

To analyse the problems considered in the project we will use, in addition to the tools of dynamical systems theory, methods coming from complex analysis, topology and probability theory.