

The question whether theorems of mathematics can be treated literally as true sentences was one of the most cumbersome puzzles from the very beginnings of philosophy. The nature of mathematics seems to deny most of our intuitions about the reality which we follow in our scientific practice. Apparently, we learn mathematical truth without gathering any empirical data, we are able to establish them with an absolute certainty, and once they are established they do not need any further verification, they obtain eternally and unconditionally. A lot of anti-naturalistic stances in the history of philosophy pointed out the existence of the mathematical truths as one of the prime motivating sources.

One of the well-established explanations to the perplexing nature of mathematics is the view that theorems of mathematics are in fact general statements. We are indeed capable of identifying uniquely a “structure” of natural numbers using exclusively sentences which refer to a certain set of objects and its subsets. Using that definition, we can for instance translate the sentence “any even number is a sum of two primes” to “in any structure which (in a precisely defined sense) *looks like* the natural numbers, if we consider the only operations which (again, in a precisely defined sense) behave like addition and multiplication, then any object which *looks like* an even number is a *quasi-sum* of two objects which *behave like* primes.

Thanks to this translation, we can explain the status of mathematical statements without postulating the existence of abstract objects, which behave radically different from the objects known to science. According to this theory, the mathematical statement would literally concern *everything*.

Although this solution seems philosophically attractive, the details of the definition what it means that certain objects *behave like* natural numbers pose tremendous philosophical problems. In order to spell out those details, we would need to use second-order logic which allows us to say that a certain property holds for an arbitrary “set” or “a group of” objects under consideration.

However, which exact statements are validities of second-order logic, depends on the accepted axioms concerning the theory of sets which also serves as the general system of axioms for the entire mathematics. It seems that we have managed to describe sentences of mathematics as sentences about everything at once only under the assumption that there exist objectively true axioms describing the notion of set. Hence, it seems that our reduction was fruitless. For at the end of the day, we still have to invoke objective truths about the abstract objects, these objects being now sets rather than numbers or operators.

On the other hand, there is a family of theorems of mathematical logic which say that a large and important class of sentences of second-order logic is not sensitive to the shape of the set-theoretic universe. There exist certain axioms which are a very natural extension to the currently accepted axiom system for sets and which imply that a significant part of second-order logic becomes true in an “absolute” manner. Although the described phenomenon is an important part of logic, it was not systematically analysed in the philosophical literature. Such an analysis would allow us to conclude whether the described phenomenon allows us to conclude that there exists an objective description of the mathematical *structures* which does not require us to postulate the existence of mathematical *entities*. In my project I intend to provide exactly such an analysis.