

## ASEP and other models of integrable probability - Abstract

Integrable probability is a theory devoted to non-equilibrium phenomena, positioned at the interface between probability and statistical physics. Integrable probabilistic systems are characterized by two features: (1) concise and exact formulas for probability distributions of a variety of interesting observables of the systems; (2) asymptotics of the systems and exact formulas provide access to descriptions of universality classes. In recent years considerable efforts are devoted to models in so-called KPZ universality class.

Kardar, Parisi and Zhang in 1986 proposed a (KPZ) model of surface growth, which appears to be a universal object that describes asymptotic fluctuations of interacting systems of statistical mechanics with space-time dependencies. Description of models in the KPZ universality class is one of the most important issues of modern probability. The current knowledge of the problem is not sufficient to tackle it in full generality; the research concentrates on limiting properties of existing models and construction of new solvable models with plausible (invariance) properties. Our goal is to considerably develop understanding of these aspects of theory in three prominent types of models: asymmetric simple exclusion processes (ASEPs), random lattice equations models (LEMs) and random directed polymers (DPs).

1. ASEP with open boundaries is a model of randomly hopping particles on a lattice of  $n$  sites and reservoirs at boundaries. Each site is occupied by at most one particle and it may jump to the left or right neighbour site only if the target site is empty. Particles can also enter or exit the system at boundaries. The model is parameterized by intensities of jumps with the jump to the right of higher intensity than the jump to the left. The main goal is to identify asymptotics of so-called height function of the ASEP in several limiting regimes when the size of the ASEP approaches infinity. The most interesting case is the WASEP regime in which Corwin and Knizel (2023), using the Askey-Wilson (AW) representation of the stationary measure of the ASEP, derived in Bryc and Wesolowski (2017), gave the first description of the stationary measure of the open KPZ. The result was restricted to the so-called fan region of the ASEP due to limitations of the AW representation. Our goal is to extend this result to the shock region by introducing a universal tool: families of "Markov-type" moment functionals based on the Jacobi matrix of the system of AW polynomials.

2. LEMs are models defined through local deterministic mechanism of iterations on  $\mathbb{Z}^2$  applied to random arguments, with a prototype of stationarity called the Burke property. Four important examples of (ultra) discrete Korteweg de Vries and Toda LEMs have been recently identified by Croydon and Sasada, 2020. Such constructions are possible in case the iterations are governed by independence preserving (IP) maps. We plan to refer to Rosenblatt's map, which maps a random vector into a collection of independent random variables, a tool scarcely known among experts of integrable probability, for systematic approach to LEMs and their properties. Sasada and Uozumi, 2022, searching for LEMs mechanisms, discovered an intriguing connection between IP and quadrirational Yang-Baxter (qYB) maps. In view of role of YB condition in integrable probability the IP-qYB "mystery" caught attention of leading researchers in the field. We expect to solve the "mystery" through identification of a common mechanism of IP characterizations for maps from the qYB class.

3. DPs can be understood as LEMs with (special) random mechanism of iterations. We are concerned with random DPs of dimension  $1 + 1$ , which are the main models of studies among the experts in the field. The basic object is so-called point-to-point partition function defined as sum over all up-right paths from  $(0,0)$  to  $(m,n)$  of products of random weights associated with edges in the path. It is known that several such models belong to the KPZ universality class. The most prominent example is the log-gamma DP introduced by Seppalainen in 2012. The basic feature which allowed for tractability of the model is the celebrated Lukacs IP property for gamma laws. Our goal is to search for new random DPs (and their asymptotics) within the class governed by IP maps, which generalize those related to the class of four basic  $1+1$  DPs. To construct candidates for new DP-type models we plan again to exploit the Rosenblatt map.