Project Summary

Multiscale modelling of fluids and tissues

One of the most fundamental concerns in modern science is the understanding of how populations of large numbers of agents organise spatially, socially and dynamically. This project focuses on addressing issues inspired by biology, as nowadays this field offers exceptional access to abundant and precise data, enabling the development of novel and complex mathematical models. The problems we shall study are formulated in the language of nonlinear partial differential equations.

Tremendous effort is required to uncover the intricate mathematical structures of these equations. Given the practical importance of the underlying questions, ranging from medical science or chemical industry to cell biology, it is imperative that a deep conceptual understanding of the mathematical models results from our research. The role of mathematics extends beyond merely comprehending the analytical structure of those new problems. It also involves explaining the qualitative behaviour of solutions and quantifying their properties.

Among the central features of this project is its multiscale approach. The problems we shall address share a common characteristic: understanding their macroscopic behaviour accurately requires far insight into a hierarchy of finer scales, encompassing individual behaviour as well as the molecular factors influencing it. In recent years, we have been witnessing profound success in applying to problems arising in mathematical biology the tools developed in the context of fluid dynamics. Even if the underlying principles are different, it turns out that a similar analytical approach can be applied in a variety of situations. Indeed, once formulated in terms of partial differential equations, a fluid, a living tissue, or even a flock of birds, do not differ much from one another. In this project, we will seek to leverage the inherent connection between biological systems and fluid flow phenomena to shed new light on both. Specifically, we will pursue the following research directions, which currently attract a lot of attention from the applied mathematics community.

Mechanical models of cell motion: Many biological systems involve the transport and spreading of substances, such as nutrients, signalling molecules, or ions. These phenomena play a vital role in various biological processes and have significant implications for the functioning of cells, tissues, and organs. Mathematical models incorporating diffusion, advection, and convection are well established nowadays in the context of modelling living tissues. Typical top-down approaches rely on phenomenological assumptions on the underlying biomechanics. However, little emphasis has been placed on the fine relationship between these models. For instance, it is of great practical interest to build rigorous bridges between modelling approaches at different scales (individual cells vs. macroscopic densities vs. geometric cell patches).

Dynamics of polymeric fluids: Polymer molecules are long chains of repeated basic structural units (monomers), typically containing the order of 10^3 to 10^6 monomers and the presence of these long-chain molecules in a fluid can dramatically affect its macroscopic properties. In particular, polymer molecules introduce elastic properties, giving rise to a range of exotic phenomena, such as shear-thinning or rod-climbing. As polymer-fluid mixtures are ubiquitous in many branches of biology and chemistry, studying coupled models of polymers suspended in a solvent is of great interest. Additionally, these models pose numerous challenges: equally at the level of their systematic derivation, mathematical analysis, and numerical simulation. Most approaches to the mathematical modelling of polymeric fluids are based on kinetic theory, in which the behaviour of the microscopic polymer molecules is characterised in a statistical sense. Handling the wide range of modelling simplifications as well as quantitatively comparing various approaches constitutes a substantial challenge for mathematical analysis.