GEOMETRIC STRUCTURES BEHIND TENSORS
AN INFORMAL SUMMARY

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Tensors show up prominently in recent technologies, especially related to deep learning, large language models etc.: *Tensorflow* library in python, Tensor processor family from Google, etc. The cause of this popularity is the *tensor* as data format for machine learning algorithms. Apart from them, tensors are used in signal processing, image recognition and other applications. Correspondingly, they are central to more theoretical aspects of computer science, including the asymptotic complexity of many linear algebra algorithms, such as matrix multiplication, inverting matrices, computing the determinant etc.

A tensor is a generalization of a matrix. A matrix is a table of complex numbers $[a_{ij}]$ where i, j are indices, for example $0 \leq i, j \leq 2023$. An order three *tensor* is a three-dimensional analogue of a matrix, that is, it is a cube of complex numbers $[a_{ijk}]$, where i, j, k are indices. Matrices are well-understood (in fact, considered trivial) thanks to linear algebra. Contrary to the naïve expectation, the situation for tensors is drastically different[1]. For tensors it is not quite so: for example computing the rank of a tensor or its border rank are NP-hard problems; impossible to effectively solve in general. Despite this, computing the border rank of some special, very specific tensors, is and will be a focal point of research because many problems can be effectively reformulated in this language.

In this project we introduce several geometric techniques to tackle the large parameter space of tensors; the core idea of the project is to provide better geometric representations of certain tensors, which in turn allow for effective work with them. This is akin to the recent transformers technique in deep learning, but more theoretical and deeper.

[1]To understand why, the Reader may think of symmetric matrices, which are quadratic forms and are classified by ranks. In this analogy, the symmetric tensors are cubic forms, and are far from being classified.