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Connecting the dots: centrality analysis from the perspective of game theory and social choice

Description for the general public

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Sometimes people use the same word to describe completely different things. Sometimes, however, they use different words to describe the same thing. In our project, we will focus on the task of evaluating elements in complex systems. This task, under various names, has been studied in the fields of network theory, game theory, and social choice theory. The goal of our project is to discover new, unexplored connections between these domains.

In network theory, functions that assess the importance of elements are called *centrality measures*. Centrality measures are used in various applications, such as studying neural networks in the brain, the World Wide Web, financial flow networks, or criminal networks. The multitude of applications has led to the development of numerous new measures in the past 20 years, each designed to better fit specific types of networks. However, more does not always mean better. Currently, when we want to identify the most important element in a specific network, we need to select one measure from hundreds of different definitions.

Therefore, in recent years, efforts have been intensified to organize the space of centrality measures. A key issue is considered to be a good understanding of the differences and similarities between centrality measures. Typically, an *axiomatic approach* is used for this purpose, which involves proposing simple properties, called *axioms*, and examining which centrality measures satisfy them. If a measure satisfies properties that are desirable in our application, we should use that measure.

The issue of evaluating elements in complex systems is not new, and the axiomatic approach has been applied to it for years. In the field of cooperative game theory, in 1953, Lloyd Shapley proposed a method, now known as the *Shapley value*, which evaluates the importance of specific players in a cooperative game using axioms. An even older example is social choice theory, where voting rules that determine the winner or winning committee are often based on the assessment of individual candidates based on voter preferences. In each of these fields, the structure of information about the elements is different. However, do they not conceal similar concepts and ideas?

> *The goal of our project is to examine centrality measures from the perspective of social choice theory and cooperative game theory. Our project will significantly deepen the understanding of centrality measures and establish a theoretical foundation for their proper use. We will build bridges between network science, game theory, and social choice theory, which will be beneficial to all three disciplines.*

Our project will be based on our latest results that show the connections between network theory and elections and coalitional games. We illustrate them in Figure 1.

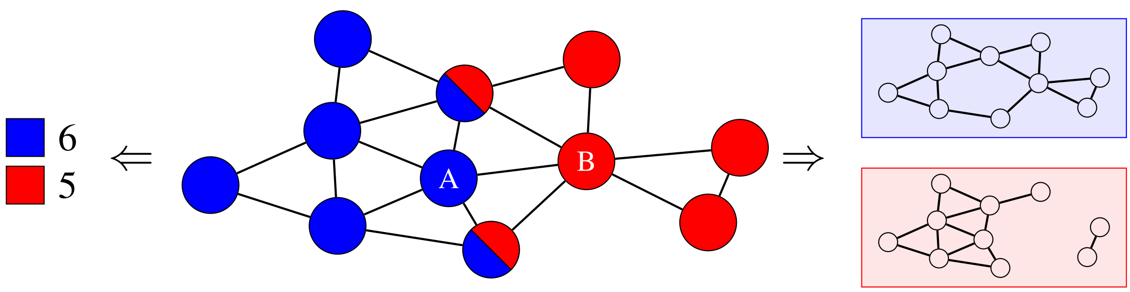

Figure 1: Who is more important – A or B? Let's vote! Every node votes for the closest of these two nodes. It turns out that the winner of such a vote is the same as the node with higher closeness centrality. Alternatively, we can look at the marginal contribution of a node to the graph, which means comparing graphs without those nodes. It turns out that vitality indices working in this way are equivalent to game-theoretic centrality measures, specifically, the Shapley value of certain games in which the nodes are players.