Logical and Philosophical Aspects of Pluralism and Potentialism in Set-Theory and Arithmetic – popular abstract

Potentialism in philosophy of mathematics is a view that universe of mathematical discourse is never given as such and fully completed, but rather unfolds gradually via steps specific to particular language or theory. The idea can be drawn back to the ancient debates concerning actual and potential infinity, where the name potentialism could be applied to an Aristotelian view that the correct sense of infinity is given by the concept of potential infinity, according to which there are no infinite sets given as complete collections, but that the task of enlarging any finite set or object (e.g. a natural number) is unbounded or indefinite. The distinction has recently come back to the attention of analytic philosophers.

The idea of potentialism recently has been applied to theories richer than arithmetic, and, thanks to development of methods of advanced set theory, such as forcing, large cardinals, inner models, set-theoretic geology etc. also found its expression in the realm of the theory of the infinite. In set-theoretic context potentialism expresses an idea that universe of entire mathematical discourse is never fully completed. This way of thinking of set-theoretic universe is closely related to the above-mentioned multiverse conception which is similarly a specific non-absolutism about sets, according to which there is no absolute or 'true' universe, but that ultimately there are rather multiple acceptable models of set theory, each instantiating a particular concept of sets.

The first and the main goal of the project is to investigate properties of various conceptions of set-theoretic multiverse from the perspective of potentialism and analyze how these two perspectives relate to each other. To this end, we plan to put the notion of potentialism to philosophical scrutiny with the purpose of specifying formal criteria of a particular set-theoretic multiverse conception falling under the scope of potentialism and examining how non-restrictive can a given mutliverse be as a potentialist system. With these criteria at hand, we are going to conduct mathematical investigations of various properties of different multiverse conceptions available in the current set-theoretic (mathematical, logical and philosophical) literature, and examine them under the potentialist perspective. This way, we plan to answer questions such as to what extent the multiverse view can indeed be seen as a potentialist one, and what light does this shed on whether the multiverse view is accurate, or to what degree the potentialist view in set-theory can be held in the light of current set-theoretic practice. By performing this task we hope to formulate possibly new set-theoretic principles that can serve as guidelines for further set-theoretic research devoted to the study of global properties of set-theoretic universe(s).

One of the tools applied recently to the study of potentialism in foundations of mathematics has been modal logic, and its model theory in particular. Relying on this work, as well as some recent work of our own, we further plan to develop this modal-logical methodology with the purpose of applying it to different multiverse conceptions in set theory, as well as to the study of a particular form of finitism related to the previous paragraph: finitism. It has been claimed that the modal-logical study of families of finite models with particular semantics reveals that infinitism or actualism is implicit in a reasonable finitist or potentialist explication of natural numbers. The purpose of the project in this respect is to put this claim to further scrutiny, via the use of mathematical tools of advanced modal logic.

The ultimate goal of the project is, using both formal (mathematical and logical) and conceptual results of the planned research, to analyze consequences of potentialism and pluralism in arithmetic (including also its finitist variant) and set theory for philosophy of mathematics, and for some aspects the infamous question of 'what is the subject of mathematics'.