Description for the general public

In this project, we are concerned with solutions of equations with integro-differential operators L on the open set U, which was formed by removing a point x_0 from a given domain D (so-called isolated singularity). Suppose we have a solution f to the equation on such a set. The main task of the project is to determine the possible behavior of the solution around the removed point x_0. Problems of this type arise naturally in many physical and geometric problems.

Under the assumption that L is the Laplace operator, the problem we shall investigate in the project, is a classic in mathematics. It has been studied by such great mathematicians as P. L. Lions, H. Brezis, L. Caffarelli, L. Nirenberg, J. Serrin , D. Gilbarg. In 2008, in one of his papers (with co-authors), L. Nirenberg wrote: "Understanding the behavior of solutions of partial differential equations near an isolated singularity is of basic importance in the study of partial differential equations."

Modern applications of mathematics necessitate the use of so-called non-local operators in many problems that have so far been considered only for local operators. Non-local operators model phenomena in which the dynamics at a given point depends on the behavior of objects at distant points. From a mathematical point of view, the specific feature of a nonlocal operator is that its value on a function u at a point depends on the value of u over the whole space. These types of operators have long been studied in quantum physics. However, the growing interest in nonlocal operators over the past two decades is due to the fact that in a large part of physical, biological, chemical and financial mathematics models, replacing classical operators with nonlocal operators in the related differential equations leads to solutions that better describe the phenomena under research (this is now supported by numerous scientific publications).

In this project, we will deal with equations with isolated singularities under dynamics given by a class of Feller integro-differential operators. We plan to prove a number of theorems that will allow us to say something about the existence or removability of singularities, the rate of blow-up of solutions around an isolated singularity, about the maximum principle for sub-solutions, and the existence of solutions with prescribed singularities. The goal of the project, however, is twofold. Firstly, we want to solve the problems formulated above, but secondly, we want to propose a new approach to the aforementioned questions by applying stochastic analysis and probabilistic potential theory. We believe that this approach will bring new results in the theory of isolated singularities even in the case of classical operators.