

Model theory is a part of mathematical logic investigating structures which are *models* of some *theory*. In this research project we want to analyse in a model-theoretic spirit a class of classical mathematical objects, namely *fields with operators*.

A *field* is an algebraic object (with addition and multiplication), such as the rationals, the real numbers or rational functions. An *operator* is a self-map of a field, which satisfies some conditions. Examples of operators are field automorphism (i. e. maps preserving the algebraic structure of a field) or derivation on the field of rational functions. Such objects are prevalent in mathematics, especially in applications of model theory to algebra and number theory, as seen in the following example:

In number theory one often considers Diophantine equations, i. e. equations whose solutions of interest are integers or rational numbers. Solving such equations can frequently be interpreted as searching for rational points on certain geometric objects, e. g. elliptic curves (which is of immense importance in cryptography) or more generally abelian varieties. Here do fields with operators spring into the action: in some cases one can interpret the set of rational points as a subset of the set of solutions (in some abstract field) of some abstract differential equations associated to the geometric object. A good understanding of fields with operators allows one to say much about this set, hence also about the solutions of the initial Diophantine equation. This good understanding of a wide class of fields with operators is the goal of this research project.

One of the tasks we want to tackle is to formulate a general framework for *iterative* operators, which would unify and generalize a few existing theories. The adjective *iterative* means that we consider operators which behave in a specific way when applied one after another, i. e. when *iterated*. We want to achieve this by using a notion from category theory (appearing also in computer science, e. g. in functional programming) - the notion of a *comonad*.

Beside that, we want also to investigate certain specific fields with free (i. e. noniterative) and iterative operators, e. g. derivations *twisted by an automorphism*. One of the basic questions we want to answer is the existence of a *model companion* of the theories of such fields. In other words, we ask whether we can axiomatize the class of all “big” fields from this class, where “big” means that all consistent sets of equations have solutions in these fields. The existence of a model companion is a fundamental question in model theory and the first step in many applications to algebra and number theory.