

The presented project concerns dynamical systems. This area of Mathematics, motivated by question coming from Physics, has been developing since the turn of 20th century. The physical motivations come mainly from two groups of problems: ordinary differential equations describing the motion of many bodies under the gravity, and statistical physics, which deals with system of many interacting bodies. A common feature of this motivating problems is the difficulty of giving a rigorous solution by simple formulas. Therefore, qualitative and probabilistic methods of describing such phenomena have been introduced.

Classical dynamical systems theory studies the evolution of a system, described by deterministic rules, i.e., the state of the system in consecutive moments depends on the state in previous moment and it is described by time invariant rules.

In the language of mathematics, we say that we are dealing with multiple composition of the same transformation with itself. Time is discrete- the successive applications of a transformation takes place in successive „seconds”.

Non- autonomous systems are also considered. These are the systems where we also perform consecutive composition but the applied map (the variation rule of the system) depends on the moment at which we transform the system.

Finally- we have also random systems, i.e. systems in which we also compose consecutive transformations, but in each step - the transformation is chosen randomly (i.e.- we draw randomly from some available collection of transformations).

The present project is concerned with dynamical systems on the plane, on a straight line, on a circle, or on more complicated metric spaces, including - the so-called complex projective spaces.

One of our goals is to study geometric properties of certain complicated invariant sets or other sets which arise naturally when studying such systems.

In particular, in the study of holomorphic dynamical systems- that is, in the study of multiple composition of a holomorphic function- a closed invariant set with very complicated geometry appears. This is the so-called Julia set. Another example of such a complicated subset described by iteration of holomorphic transformations is the so- called indecomposable continuum. It is interesting to note that to study geometric properties of such sets, one needs to use advanced methods from various fields of mathematics, among them- functional analysis and probability theory.

We also study several classes of non- autonomous and random systems, including systems where we randomly choose a transformation of a circle or a disc and study the random walk determined by this process. We look for stationary measures for this random walk and study its properties.

The question we ask in the project are of interest to many mathematicians. We expect that our answers to these questions will be interesting and stimulating to mathematicians working in similar area.