Algebraic geometry is concerned with studying properties of equations through analysis of the geometry if their solutions. Geometric objects associated with a specific equations is called *algebraic variety*; the simplest examples are line or conic but in general these objects are very complicated.

Fundamental goal of my project is the analysis of *one-parameter families* of algebraic varieties. One can think of them as of collections of equations with a parameter $t$. For example, for positive real numbers $t \in \mathbb{R}_{>0}$ equations

$$x^2 = y^2 = t$$

describe a family of circles with changing radius.

In particular the subject of my investigation will be families of three-dimensional Calabi-Yau manifolds. They form a special class of algebraic varieties which posses a complex volume form $\omega$. Motivation for studying them comes both from physics (string theory), as well as arithmetic (one-dimensional Calabi-Yau varieties were an important tool in the proof of Fermat's Last Theorem).

For a given family of Calabi-Yau threefolds $X_t$ (with parameter $t$) each element posseses a complex volume form $\omega_t$. It turns out that integrals $_{\gamma_t}\omega_t$ satisfy a fourth order differential equation, called the *Picard-Fuchs equation* of the family $X_t$. Main subject of my project is extracting information about the family from the knowledge of its differential equation.

The basic tool used in studying differential equations is its *monodromy group*. It described how the solutions change when we move them along a loop omitting singularities. In the case of families of Calabi-Yau threefolds the monodromy group is a subgroup of the symplectic group $\mathrm{Sp}(4, \mathbb{Z})$, which allows one to study its arithmetic properties. This is one of the goals of my project. Another one is to study monodromy matrices in certain local bases, so called *Frobenius bases*.

In this context special values of $L$-functions naturally appear. $L$-function is a complex function connected with certain modular form $f$ and special values $L(f, 1)$, $L(f, 2)$ naturally appear as the values of solutions of the Picard-Fuchs equation at certain singular points. (A *singular point* is a value of the parameter $t$ for which the element $X_t$ degenerates; in the example of family of circles the singular point would be $t = 0$, where circle reduces to a point). Study of the connection between the Picard-Fuchs equation and modular forms is the second of my research goals.

Connection between modular forms and Calabi-Yau threefolds is deep and still not fully understood. One way it can be described is that the number of solutions of the equation defining a Calabi-Yau threefold as a congurence modulo a prime number $p$ is encoded in the $L$-function of certain modular form. The last goal of my research project is to find a good model of the considered variety for primes $p$ for which the equation defining the variety is singular. It should lead to a better understanding of the Euler factors of the $L$-functions corresponding to those primes.