Various notions of convergence in partial differential equations

In partial differential equations, one is often interested in limiting procedures between two different (either in assumptions, or in form, or both) systems. In the study of singular limits, one can ask himself what kind of information is transported together with convergence between systems. Point-wise, uniform, strong, weak convergences all give different qualities to the functions which is a limit of the given sequence. Therefore, it is important to establish what type we are dealing with. Similarly, the question of stability of a system is one of the most classical and major considerations in partial differential equations; it tries to give an answer to a well-posedness aspect of the equation: if I take a sequence of solutions and consider their limit, is it still a solution? We are interested in expanding this long line of research and answer those questions with regard to the problems arising in fluid mechanics. In particular, we are interested in the two-dimensional Euler equations in a bounded domain and viscoelastic, heat conducting fluids (Giesekus, or more general type).

The $2D$ Euler equations have been studied extensively throughout the years, but the main research has been focused on the periodic domains or on the whole $\mathbb{R}^2$ space. This is because the boundary conditions for the velocity of the fluid do not give any information on the boundary conditions on its vorticity (here, vorticity is a quantity, which gives information on how much a fluid is "spinning"). We would like to expand the state-of-the-art to the territory that has hardly been brushed by a scientific community. Our aim here is to expand on the ideas given by Lopes, Seis and Wiedemann, as well as Constantin and Vicol in their latest papers, by researching the inviscid limit of Navier-Stokes equations in two-dimensional bounded domain, and for unbounded vorticity. The significance of such research lies in a simple fact, that the flow of the fluid in the real world is often restricted by physical boundaries, and the rate of its "spin" can become unbounded.

Likewise, the theory of viscoelastic fluids has a long history, with the first model being developed by Burgers in 1939 (which can be viewed as a mixture of two non-Newtonian fluids), but only as of late, the researchers were able to organize the hierarchy of the generalizations of the classical Burger-type models, which made their analysis that much easier. The importance of those considerations lies in a vast landscape of substances described by them; the classical examples of materials, the behavior of which is governed by the models mentioned above, are geomaterials, such as asphalt, biomaterials like vitreous in the eye, or synthetic rubbers. We would like to study a few of those generalizations and the behavior of the so-called weak solutions to them. In principle, such solutions satisfy some kind of an integral equality, instead of differential one, which means that the searched for functions, do not need a high regularity. This helps us treat the cases of mixtures of fluids, such as viscoelastic-rate type models. We are mostly interested in the questions of well-posedness: existence and long-time stability, but do not want to restrict ourselves in the considerations.