AUGMENTED ORIENTATIONS AND EFFECTIVE CONSTRUCTIONS FOR ALON-TARSI METHOD

Tertium non datur – the law of excluded middle[1], used frequently in mathematical proofs at least since Aristotle, has been contested by some logicians in the beginning of XX-th century as the main source of *nonconstructivity* in mathematics. A proof is constructive when besides proving that some object exists, it provides some clues for constructing such an instance. Original motivation for questioning nonconstructive arguments had philosophical provenience. The idea of constructive mathematics was not widely approved in the scientific community. One of the most prominent mathematicians of that time, David Hilbert strongly opposed it:

Taking the principle of excluded middle from the mathematician would be the same, say, as proscribing the telescope to the astronomer or to the boxer the use of his fists. (Hilbert [1928])[2]

Nearly hundred years later the perspective is quite different. One of the sources of that change stems from application of mathematical tools in computer science. When an engineer needs to develop a chip architecture that satisfy some constraints, formal proof that such a design is possible is of little use to him. Clearly, some sketch of a design or an arrangement of building blocks would be more helpful. One could argue that finding an object that is otherwise known to exists is an easy task – the number of possibilities is usually limited and one can use a computer to scan through all of them to find a satisfactory one. However, for most of practical problems it would take ages for modern computers to run such a search. We not only need algorithms that are capable of finding solutions, we need them to be *efficient*. That notion is well approximated by the theoretical concept of algorithms with polynomial running time.

In 2002, on the International Congress of Mathematicians, Noga Alon described algebraic and probabilistic methods as the two main techniques that "played a crucial role in the development of modern combinatorics" (i.e. the field mathematics that studies finite structures). Interestingly both these tools are in principle non-constructive. A lot of work has been put in transforming probabilistic proofs into effective constructions. Recent decades brought significant successes in that area and greatly enhanced our understanding of the role of randomness in computation. Building algorithms for the proofs relying on the algebraic techniques are quite often elusive.

Our project aims at particular application of the algebraic method to graph coloring problems known as Alon-Tarsi method (graph coloring is a versatile framework for expressing algorithmic tasks). We are going to analyze known nonconstructive proofs, devise algorithms and study obstacles and limits of efficient algorithmization. We are mostly interested in applications for two well established and deeply researched areas. First, the list coloring of planar graphs, their variants, and generalizations to other graphs with relatively small number of edges. Second, the list coloring of edges of graphs. One of the great successes of the theory of computational complexity is development of tools for proving that some problems are impossible to be effectively solved by a computer. We expect to meet these kind of barriers so it is likely that the outcome of the project will also contain results of this kind.

[1] *For every proposition, either this proposition or its negation is true.*

[2] source *The Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/