Reverse mathematics is a research programme in mathematical logic that has been developed since the 1970's. Its aim is to understand what sort of axioms about the existence of complicated infinite sets, uncomputable functions, and other abstract objects are needed to prove specific mathematical theorems. The point is that the theorems themselves should belong to "ordinary mathematics": they should not be very advanced statements of abstract set theory, but rather everyday results that one can encounter, for example, during the course of undergraduate studies. To show that a given axiom is needed to prove a theorem, one proves that *the theorem implies the axiom* – hence the name "reverse mathematics". Discovering which axioms are really essential for the proof of particular theorems is highly interesting from a philosophical point of view, since it helps us to understand the logical structure of mathematics as a whole. However, it may also have some applications, for example by telling us that some object can be found by an effective computational procedure or that some result has a hitherto missed elementary proof. The area has by now attracted enough attention to become the topic of a book written for a general audience: John Stillwell's *Reverse Mathematics. Proofs from the Inside Out*, published in 2018.

In the history of reverse mathematics, one can roughly distinguish two periods. During the first period, it was shown that many theorems from all areas of mathematics are equivalent to one of just a few typical set existence axioms that could be put into a linearly ordered hierarchy in terms of strength. Then, roughly since the 1990's, logicians began to discover many theorems that are not equivalent to any of the typical axioms, especially among results from combinatorics. A particularly famous example of such an "anomalous" result is Ramsey's theorem for pairs and two colours, the statement that any infinite graph will contain an infinite subgraph in which all pairs of vertices are connected by an edge or an infinite subgraph in which there are no edges.

One way to understand the logical strength of a theorem, or of an axiom, is to study its *first-order consequences*, that is, the set of statements about finite objects (e.g. natural numbers) that it implies. For example, an extremely important set existence axiom is Weak König's Lemma, which is officially a statement about infinite trees, but it is equivalent to many theorems across mathematics, such as the fact that if $[0, 1]$ is contained in the union of infinitely many open intervals, then it is also contained in the union of some finite number of them, or various theorems on the existence of solutions of differential equations or the existence of certain ideals in rings. It is known that, in general, it is not possible to prove Weak König's Lemma in an algorithmic way, that is, sometimes the objects that exist thanks to the lemma have to be uncomputable. On the other hand, however, every statement about natural numbers provable using Weak König's Lemma has an elementary, "purely computational" proof.

In a paper published in 2018, L. Patey and K. Yokoyama showed that also simple enough statements about natural numbers that can be proved using Ramsey's theorem will always have "computational" proofs that avoid using the theorem. However, giving a full description of the first-order consequences of Ramsey's theorem is a notorious open problem. It is known that to solve this problem, one has to gain a better understanding of *nonstandard models of arithmetic*: structures that satisfy most of the usual properties of the set of natural numbers but also contain additional numbers that are infinite viewed from outside but look just like any other natural number from the point of view of the structure.

A problem that currently deserves particular attention is to characterize the ways in which nonstandard models of a certain special kind, satisfying the so-called collection principle, can be extended so as to avoid destroying their similarity to the usual, standard natural numbers. Recently, we have shown that the possibilities of extending models of collection are more limited than for other kinds of nonstandard models, but the information we currently have is not yet sufficient to determine whether models of collection can always be extended in a way that will make them satisfy Ramsey's theorem. In our project, we will study various questions about the structure of nonstandard models – not only about their extensions of various kinds, but also about initial segments. However, understanding nonstandard models will not be an end in itself, but a tool in the study of questions about the existence of proofs, such as whether a given theorem (usually from combinatorics) can be used to prove a certain axiom or what sort of first-order consequences the theorem has.