Mathematical control theory in problems arising in flow-structure interactions

The proposed work lies in the general area of partial differential equations (PDE) and applied analysis. More specifically, it is a comprehensive investigation of a well-known problem in theoretical aeroelasticity: understanding the oscillations of a thin flexible plate interacting with an inviscid potential flow in which it is immersed. Applied analysis along with several deep mathematical questions are the cornerstones that provide the quantitative link between mathematics and other (engineering, medical) sciences. The main purpose of the project is to develop a mathematical methodology which will be suitable for rigorous analysis of dynamics governed by flow-structure interactions including the somewhat related free and moving boundary problems. This area, due to the challenge of both the mathematics involved as well as a direct relation with technology, physical and medical sciences, remains a timely topic of great interest.

Oscillations and turbulence resulting from flow-structure interactions are ubiquitous in nature. Illustrations include an oscillating bridge or a building exposed to strong winds; the wing of an airplane immersed in a gas/air; or a facial palate subjected to an uncontrolled flow intake (treatment of apnea). In all these cases, one deals with a structural dynamical instability called "flutter". Flutter consists of sustained oscillations which may cause severe damage of the materials' properties and eventual collapse of the structure. The Tacoma bridge or the NASA Helios mishap are classical examples. It motivated the need for developing a rigorous theory for its prediction. A preliminary great challenge or obstruction consists of lack of possession of a correct modelling in agreement with the experiment. The critical need for predicting and suppressing flutter has provided strong motivation for both experimental and computational research in engineering and theoretical research in mathematics. Understanding and describing mathematically air-loads affecting wings and airfoils and analyzing the resulting time-dependent motions has became a major areas of research. Selected but important advances have been achieved recently. Particularly at the level of subsonic velocities of the flow and very basic boundary conditions imposed on a structure. Even though the study of the flutter has been originally motivated by aeronautical applications, the sectional and preliminary common has emerged in many areas of civil engineering involving flows, bridges and tall buildings; in mechanical engineering on flows about turbomachinery blades and fluid flows in flexible pipes; in nuclear engineering on flows about fuel elements and heat exchanger vanes. Moreover, aeroelasticity plays a crucial role in the development of new aerospace systems such as unmanned air vehicles UAVs.

Above we have pointed out the destructive aspects of flutter. However, it must be realized that flutter can also have some beneficial effects in harvesting of the energy. Illustration include: wind mills harvesting energy from an airflow; water converters -floating bodies, harvesting energy from water waves hitting the structure. Thus, the problem of understanding and "controlling" flutter is fundamental in many areas of real life. More specifically, understanding the onset of flutter is critical to either preventing and avoiding the destructive effects of flutter or else inducing beneficial effects such as harvesting energy in ecological studies. This also explains that flow-structure models, and their analysis, have attracted considerable attention in the past literature. However, the vast majority of the work on flow-structure interactions is devoted to either numerical studies or experimental studies. This is not surprising, taking into considerations on one side multitude of engineering applications aiming at flutter suppression or its enhancement and on the other side challenges related to mathematical modeling of this phenomenon.

The main aim of the presented project is to provide a mathematical treatment of flutter phenomena which is based on continuum modeling described by a system of nonlinear PDE coupled at an interface. This will involve studying the interaction between linearised (unstable) Euler equations and a nonlinear structure accounting for large displacements. Mathematical analysis of the problem is much less costly than trial and error based computational or experimental studies. On the mathematical side, the proposed work will involve an analysis and control of a coupled nonlinear PDE system with an interface. On the other hand, these are are problems related to the modeling of phenomena that are extremely important from a practical point of view The study will include aspects of existence, uniqueness, robustness of solutions and their long time behavior, control and stability.