After the parameter estimation, we must always ask: *what about the quality of the estimator's outcome so one can have confidence in this outcome*? In the case of precise GNSS positioning models, validation is critical to evaluate the quality of positioning results. The most undesirable is accepting the incorrect solution with statistics indicating its high accuracy. Thus, it is more undesirable than simply rejecting the correct solution with low accuracy. These two scenarios indicate that quality control is undoubtedly relevant because the statistical properties of positioning results cannot be assessed without reliability. For this reason, the last step of the adjustment framework – validation of the parameter estimates - should be investigated for regularized GNSS models because validation procedures are extremely limited for such models. There is a lack of conclusive studies that provide advanced quality control statistics considering the regularization principle in a testing procedure.

Due to the lack of visible satellites with high-quality signals, poor geometric construction of the positioning, satellite ephemeris errors, and uncounted atmospheric delays, the strength of precise GNSS positioning models degrades, especially in harsh observing conditions, e.g. urban canyons, forests, or during geohazards monitoring in mountain areas. The outliers in the code pseudoranges, cycle slips in the carrier phases, and multipath effects occur, leading to the deterioration of the positioning accuracy. Thus, the availability of new satellite signals does not necessarily correspond to higher accuracy. Therefore, precise positioning models are poorly conditioned under these environments and identified as ill-posed 'weak' models. Since choosing a proper math model is complicated in the case of ill-posed GNSS problems, regularized models have been increasingly chosen, especially when modelling an ultra-short session such as a single-epoch data sample. The regularized GNSS models can be provided by Tikhonov regularization. Thanks to the improved precision, but at the cost of slight regularized bias, the regularized least-squares (R-LS) estimates of the parameters of interest have enhanced accuracy in the sense of mean squared error (mse). Despite losing the estimator's unbiased nature, the R-LS solution is considered stable because the regularization parameter (RP) (or matrix) balances the contributions of improved precision and bias in the solution's mse. The dispersion of R-LS estimates is reduced around the true parameter values. Thus, the probability density function (PDF) of the R-LS estimator is more peaked. Unfortunately, the mean of this estimator is biased, making the probability distribution non-central. This non-centrality is hard to assess and constitutes the cost of regularizing weak GNSS models. On the other hand, it creates a serious problem for variance component estimation (VCE) and then quality control. The R-LS estimator, in fact, its solution, propagates the biasedness on regularized measurement residuals. Thus, the R-LS residuals are biased, and quality control statistics constructed based on these residuals will not follow the assumptions underlying the current statistical hypothesis testing theory. Thus, one cannot simply apply the DIA (Detection-Identification-Adaptation) procedure to validation in regularized models.

This project proposes enhancing the adjustment framework of regularized precise GNSS positioning models to be applied in Earth sciences when harsh observing conditions occur. The main goal is to evaluate the current statistical hypothesis testing theory of the quality control, thus developing a new rigorous, with optimality properties, validation approach of regularized solutions to ill-posed GNSS models. Therefore, the quality of regularized solutions can be assessed thanks to the advanced validation procedure that considers the regularization principle. The project combines advanced adjustment computation theory and methods with advanced GNSS data processing methods. The research tasks are conducted based on the following research plan:

1. Specification of ill-posed GNSS models
2. Analysis of the Tikhonov regularization for an ill-conditioned GNSS parameter estimation
3. Evaluation of the current statistical hypothesis testing theory of the quality control
4. Development of a new rigorous validation approach of regularized solutions to ill-posed GNSS models
5. Numerical experiments through methods of random Monte Carlo simulations and real GNSS data
6. Conclusions and recommendations

Considering the state-of-the-art literature and being aware of the extremely limited studies on the validation in regularized models, the current project's novelty can be assessed as high. It shall be emphasized that this project will provide an important outcome that can be valuable for different GNSS applications in geosciences. Concerning the hypothesis testing theory, the project results can shed a new light on validating the regularized solutions of ill-posed GNSS models.