The theory of elliptic partial differential equations is a very rich and well-understood mathematical theory. Boundary value problems for classical elliptic differential equations have many applications in e.g. electrostatics, continuum mechanics, hydrodynamics, diffusion theory, optimal transportation, geometric optics or non-equilibrium statistical physics.

In the last 25 years nonlocal equations for fractional Laplacians and other nonlocal operators have been extensively studied by mathematicians both from the PDE and the Probability communities. The theory of nonlocal equations has real world applications in e.g. quantum mechanics, image recognition, statistical mechanics, meteorology and finance.

Recently there is a great interest in nonlocal anisotropic elliptic operators. These operators appear in the natural way as generators of solutions of stochastic differential equations driven by jump Lévy processes in R^d with independent components. Applications of such stochastic equations to statistical mechanics, to fluid mechanics and to finance are very well described in the book by S. Peszat and J. Zabczyk “Stochastic partial differential equations with Lévy noise”.

The aim of our research project is to study existence, uniqueness and properties of solutions of boundary value problems for nonlocal anisotropic elliptic equations on domains in R^d. In particular, we are interested in estimates of solutions of the above boundary value problems and in estimates of derivatives of solutions. We plan to study Dirichlet heat kernels, Green functions and Poisson kernels for nonlocal anisotropic elliptic operators on various domains in R^d. We plan also to study the Schrödinger equation based on a relativistic Hamiltonian with confining potential. This Hamiltonian is an example of a nonlocal anisotropic elliptic operator. Such Hamiltonian appears in Lieb’s and Seiringer’s book “The stability of matter in quantum mechanics”.

In the recent years we studied properties of solutions of stochastic differential equations driven by jump Lévy processes in R^d with independent components. These results will help to study boundary value problems for nonlocal anisotropic elliptic equations in domains in R^d. It follows from our previous results that due to the anisotropy of the operators (for some choices of coefficients) the corresponding heat kernels have some unexpected features not seen in the classical theory. We expect that similar effect may hold for Dirichlet heat kernels for some choices of coefficients and domains.

Our ultimate goal is to contribute to building the theory of nonlocal elliptic equation similar to the classical theory of elliptic differential equations. This research project is an important step toward this goal.