Martingales and Multilinear Operators
DESCRIPTION FOR THE GENERAL PUBLIC

Martingales form a class of stochastic processes which is of fundamental importance to the theory of stochastic integration, both in the discrete and continuous-time setting. This theory in turn is a basic tool in applications, including stochastic modelling and financial mathematics, it also plays a crucial role in other research fields, such as harmonic analysis and functional analysis. From the viewpoint of these applications, it is often of interest to have tight a priori estimates for various objects intricately associated with martingales (e.g., maximal functions or square functions). Such a control is important for theoretical reasons, for example, it enables the use of limit theorems and guarantees that appropriate quantities are well-defined. The subfield of martingale theory, devoted to the study of various estimates, has been an independent and very active research area for almost 100 years now.

The purpose of the project is to investigate the fruitful interplay between the theory of martingales, or rather martingale-like processes, and the theory of operators appearing naturally in harmonic analysis, including maximal functions and singular integrals. The analytic context, in which the latter operators are linear, has been studied in depth and the interconnections are considered to be well understood now. The main emphasis of the planned research will be put on the multilinear case, for which the aforementioned interplay awaits to be explored; in particular, the goals of the project include the development of new techniques available in the area. The theory of multilinear operators is a relatively new research field, motivated by applications to elliptic and dispersive partial differential equations, ergodic theory and complex function theory, among others. The first works in this direction appeared around twenty years ago, the field has been intensively developed in the last decade.

The research will also be conducted in the more general noncommutative setting, in which functions (to which the multilinear operators are applied) are replaced with elements of a given von Neumann algebra. There are two important features associated with this extension. One of the major problems is the small number of technical tools which can be used in the study of various estimates: most of pointwise or trace inequalities become false when passing from the classical to the non-classical case. This often disables the direct transfer of the analysis and requires inventing clever techniques, thus making the study of various estimates much more challenging and fascinating. On the other hand, the passage from the commutative to noncommutative setting often reveals certain unexpected phenomena, e.g., certain quantities become incomparable; certain classical objects can be generalized to the noncommutative realm in many plausible ways (so there is no unique natural extension); the constants in certain estimates behave in a completely different manner after the passage to the noncommutative case, which exhibits some structural obstacles in operator theory.

The project assumes the study of four research topics which are important from the viewpoint of the further development of the area. These include the study of multilinear Muckenhoupt weights - important positive functions satisfying appropriate balance conditions; the investigation of selected weighted estimates for multilinear maximal operators and singular integrals; the inspection of the multilinear interplay between differentially subordinate martingales and Lévy multipliers; and finally, the study of noncommutative multilinear operators.